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The Temperature Concept from the Macroscopic Point of View

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IN textbooks of college physics, the temperature concept is usually introduced by an appeal to the sense perception of hotness and coldness. Thermal equilibrium is defined as equality of temperature and is accepted as a fact of experience. The postulate, "two bodies each in thermal equilibrium with a third are in thermal equilibrium with each other," is stated as a fact of experience but plays no role in the logical development of the subject. The fact that some bodies have convenient properties whose values depend on the temperature is accepted also as the result of experience. Such a body is called a *thermometer*, and a set of rules is formulated whereby measurements of this temperature-dependent property are combined to give a number designated as the centigrade (or, as the case may be, Fahrenheit) temperature.

This procedure is quite adequate in elementary physics and there can be no serious objection to it. In intermediate and advanced physics, however, its adoption is open to the objection that the temperature rests upon anthropomorphic and intuitive ideas. For a proper understanding of the role of temperature in thermodynamics and in statistical mechanics, it is worth while to establish as objectively as possible the existence of temperature as a property of all macroscopic systems. This was first accomplished by Carathéodory[1] who inferred the existence of a temperature function from a set of equations in a formal and abstract manner that will be described in a later portion of this paper. In most intermediate and advanced books, the method of Carathéodory either has been disregarded or has been copied almost verbatim without simplification or elucidation. It is the purpose of this paper to bridge the gap between the anthropomorphic simplicity of the elementary treatment of temperature and the difficult formalism of Carathéodory in a manner that the authors hope will be acceptable to teachers of intermediate physics.

[1] C. Carathéodory, Math. Ann. **67**, 355 (1909).

The system

From the macroscopic point of view, a thermodynamic system at rest may be described in terms of its composition, mass, and one or more *pairs* of quantities, such as pressure and volume, tension and length, surface tension and area, magnetization and magnetic field intensity, electric charge and potential difference, and the like. If the system is heterogeneous, such a description is required for each of the homogeneous parts. To simplify our discussion, we shall deal only with systems of constant mass and composition, each requiring only one pair of coordinates for its description. This involves no essential loss of generality but a considerable saving of words.

The work done during a change of volume dV under a hydrostatic pressure P is $P\,dV$. Similarly,

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when a wire is stretched an amount dL under a tension F, the work is FdL. In the case of all the aforementioned pairs of quantities, one member is of the nature of a generalized force, whereas a change in the other is a generalized displacement. In referring to any nonspecified system, we shall use the symbol Y to refer to the generalized force, and X for the quantity whose change is the generalized displacement.

Thermal equilibrium

Any state of a system in which Y and X have definite values that remain constant so long as the external conditions are unchanged is called an *equilibrium state*. We find that the existence of an equilibrium state in one system depends on the proximity of other systems and on the nature of the wall separating them. Walls are said to be either *adiabatic* or *diathermic*. If a wall is adiabatic, states Y, X for system A and Y', X' for system B may coexist as equilibrium states for any attainable values of the four quantities, provided only that the wall is able to withstand the stress associated with the difference between the two generalized forces, Y and Y'. If the wall is diathermic, the values of Y, X and Y', X' will change spontaneously until an equilibrium state of the combined system is attained. The two systems are then said to be in *thermal equilibrium* with each other.

The zeroth law of thermodynamics

Imagine two systems A and B separated from each other by an adiabatic wall but each in contact with a third system C through diathermic walls, the whole assembly being surrounded by an adiabatic wall. Experiment shows that the two systems will come to thermal equilibrium with the third, and that no further change will occur if the adiabatic wall separating A and B is then replaced by a diathermic wall. If, instead of allowing both systems to come to equilibrium with C at the same time, we first have equilibrium between A and C, and then equilibrium between B and C (the state of system C being the same in both cases) then, when A and B are brought into communication through a diathermic wall, they will be found to be in thermal equilibrium. We shall use the expression, "two systems are in thermal equilibrium," to mean that the two systems are in states such that, if the two *were* connected through a diathermic wall, the combined system would be in thermal equilibrium. These experimental facts may then be stated concisely in the form: *Two systems in thermal equilibrium with a third are in thermal equilibrium with each other*. Following R. H. Fowler, we shall call this postulate the *zeroth law of thermodynamics*.

Method of Carathéodory

According to Carathéodory, thermal equilibrium between system A (coordinates Y, X) and system C (coordinates Y'', X'') implies a relation among the four coordinates which may be written in the form

$$f_{AC}(Y, X; Y'', X'')=0,$$

whereas, thermal equilibrium between system B (coordinates Y', X') and system C implies the relation,

$$f_{BC}(Y', X'; Y'', X'')=0.$$

From the zeroth law, we have then

$$f_{AB}(Y, X; Y', X')=0.$$

It is maintained by Carathéodory that these three relations can be satisfied when, and only when, the functions are of such a character that

$$f_{AC}(Y, X; Y'', X'')=t_A(Y, X)-t_C(Y'', X'')=0,$$
$$f_{BC}(Y', X'; Y'', X'')=t_B(Y', X')-t_C(Y'', X'')=0,$$
$$f_{AB}(Y, X; Y', X')=t_A(Y, X)-t_B(Y', X')=0;$$

whence,

$$t_A(Y, X)=t_B(Y', X')=t_C(Y'', X'').$$

The functions t_A, t_B, t_C are called the *temperature functions* of systems A, B, C, respectively, and are seen to be equal when thermal equilibrium is established.

As far as the authors are aware, neither Carathéodory nor any other author[2] has given a proof of this theorem, and it is doubtful whether a general proof can be given without making further assumptions. Therefore, it seems worth while to approach the matter in a simpler manner in order to arrive at the concept of temperature in a less formal way.

[2] M. Born, Physik. Zeits. **22**, 218 (1921); A. Landé, *Handbuch der Physik*, vol. ix, p. 281; S. Chandrasekhar, *Introduction to the study of stellar structure* (Univ. of Chicago Press).

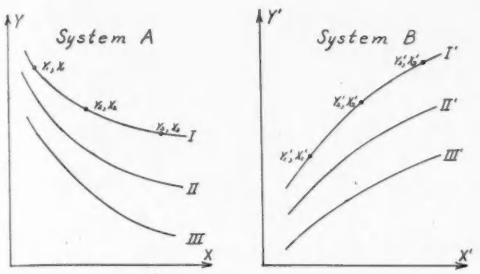

FIG. 1. Isotherms for two different systems.

Existence of isotherms

Consider a system A in the state Y_1, X_1 in thermal equilibrium with a system B in the state Y_1', X_1'. If system A is removed and its state changed, there will be found another state Y_2, X_2 that is in thermal equilibrium with the *original* state Y_1', X_1' of system B. Experiment shows that there exists a whole set of states—Y_1, X_1; Y_2, X_2; Y_3, X_3—any one of which is in equilibrium with this *same* state Y_1', X_1' of system B, and all of which, by the zeroth law, are in thermal equilibrium with one another. We shall suppose that *all* such states, when plotted on a $Y-X$ diagram, lie on a curve such as I, Fig. 1, which we call an *isotherm. An isotherm is the locus of all points representing states in which a system is in thermal equilibrium with one state of another system.* We make no assumption as tc the continuity of the isotherm, although experiments with simple systems indicate usually that at least a portion of an isotherm is a continuous curve.

Similarly, with regard to system B, we find a set of states—Y_1', X_1'; Y_2', X_2'; and so on—all of which are in thermal equilibrium with *one* state (Y_1, X_1) of system A, and therefore in thermal equilibrium with one another. These states are plotted on the $Y'-X'$ diagram of Fig. 1 and lie on the isotherm I'. From the zeroth law, it follows that all the states on isotherm I of system A are in thermal equilibrium with all of the states on isotherm I' of system B. We shall call curves I and I' *corresponding isotherms* of the two systems.

If the experiments just outlined are repeated with different starting conditions, another set of states of system A lying on curve II may be found, every one of which is in thermal equilibrium with every state of system B lying on curve II'. In this way, a family of isotherms, I, II, III, and so forth, of system A, and a corresponding family, I', II', III', and so forth, of system B, may be found. Furthermore, by repeated applications of the zeroth law, corresponding isotherms of still other systems C, D, and so forth, may be obtained.

The temperature concept

All states of corresponding isotherms of all systems have something in common, namely, that they are in thermal equilibrium with one another. The systems themselves, in these states, may be said to possess a property that insures their being in thermal equilibrium with one another. We call this property *temperature. The temperature of a system is a property that determines whether or not a system is in thermal equilibrium with other systems.*

The scalar character of temperature may be established on the basis of the zeroth law of thermodynamics. For systems A and B to be in thermal equilibrium, all the information that is needed is that both A and B are in thermal equilibrium with C. This is not true for mechanical equilibrium of elastic crystalline bodies; the tensor character of the stresses of such bodies arises from the fact that two crystalline bodies need not necessarily be in mechanical equilibrium with each other just because each is in mechanical equilibrium with a third body.

Since temperature is a scalar quantity, the temperature of all systems in thermal equilibrium may be represented by a number. The establishment of a temperature scale is merely the adoption of a set of rules for assigning one number to one set of corresponding isotherms, and a different number to a different set of corresponding isotherms. Once this is done, the necessary and sufficient condition for thermal equilibrium between two systems is that they have the same temperature. Also, when the temperatures are different, we may be sure that the systems are not in thermal equilibrium.

Establishment of a temperature scale

To establish an empirical temperature scale, we select some system (coordinates Y, X) as our standard thermometer and adopt a set of rules

for assigning a numerical value to the temperature associated with each of its isotherms. To every other system in thermal equilibrium with the thermometer, we assign the same number for the temperature. The simplest procedure is to choose any convenient path in the $Y-X$ plane, such as that shown in Fig. 2 by the dashed line $Y=Y_1$. The values of X, or of some convenient function of X, where each isotherm intersects this line may then be taken as the temperature. This procedure requires that the standard thermometer be so chosen that the portions of its isotherms that are intersected by the line $Y=Y_1$ be continuous, monotonically increasing or decreasing functions of X; otherwise, there might be two or more different values of X corresponding to the same isotherm.[3]

Assuming that our thermometer satisfies these requirements, we choose as the simplest function of X, a linear function; thus

$$t(X)=aX+b, \quad (Y=Y_1).$$

It follows from this choice of the form of the function that equal temperature intervals correspond to equal changes in the value of X. If $t(X_s)$ denotes the value of the temperature when the standard system is in thermal equilibrium with steam condensing under a pressure of one atmosphere, then

$$t(X_s)=aX_s+b.$$

Similarly, the temperature of ice melting under a pressure of one atmosphere is given by

$$t(X_i)=aX_i+b.$$

We may now determine a by arbitrarily choosing 100 equal temperature intervals (centigrade degrees) between the steam point and the ice point

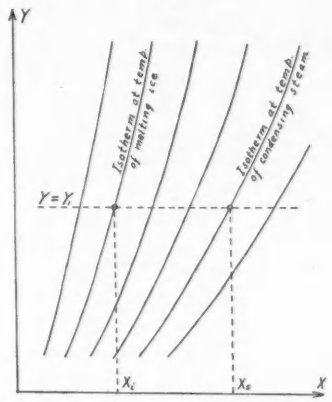

FIG. 2. Method of assigning numerical values to an arbitrarily chosen standard system, or thermometer.

—the arbitrarily chosen standard fixed points. Thus

$$t(X_s)-t(X_i)=100,$$
$$aX_s+b-(aX_i+b)=100,$$
$$a=100/(X_s-X_i).$$

To determine b, we arbitrarily assign $t(X_i)=0$, whence

$$b=-100X_i/(X_s-X_i),$$

and

$$t(X)=100(X-X_i)/(X_s-X_i) \quad (Y=Y_1).$$

It should be emphasized that "higher" temperature or "lower" temperature has no meaning until a scale is set up. It is entirely arbitrary that the number associated with the temperature of steam is larger than that associated with the temperature of ice. The converse could have been chosen, and indeed was chosen in the original scale of Celsius.

The set of rules for measuring temperature embodied in the centigrade scale gives rise to numbers that are found to depend on the nature of the system chosen as the thermometer. Consequently, the next step is to establish a set of rules which, when applied to a whole class of systems, will give rise to the same number. If we choose any gas as a standard system, we know that, in the limit as the pressure approaches zero, the isotherms are described by Boyle's law,

$$\lim_{P\to 0}(PV)=\text{const.}$$

A method of assigning a number to each isotherm

[3] Water in the neighborhood of 4°C is an example of a system that cannot readily be used for thermometric purposes in spite of apparent conformity to the requirements laid down here. If we use P and V as coordinates, we find that isotherms corresponding to different temperatures have points of intersection. One such intersection point is $P=1A_n$, $V=1.000003$ ml/g, which corresponds to either 2° or 6°C. The essential difficulty here is that we had assumed that the two coordinates P, V were sufficient to specify a state. The crossing of the isotherms on a $P-V$ diagram, however, shows that a third quantity, which is presumably of the nature of a degree of molecular association, has to be specified. If it were possible to determine and control this additional quantity, it would be possible to formulate a set of rules whereby water could be used for thermometric purposes.

may therefore be adopted which will provide the same set of numbers, no matter what gas is used. This method, which defines the so-called *absolute gas scale*, requires that the number assigned to any isotherm be proportional to the limiting value of PV for that isotherm. Denoting the absolute temperature by θ, we have

$$\theta = a \lim_{P \to 0} (PV),$$

where a is determined from the condition that there shall be 100 degrees between the standard fixed points, or

$$\theta_s - \theta_i = 100.$$

Calculating a, we find

$$a = \frac{100}{\lim (PV)_s - \lim (PV)_i},$$

and

$$\theta = 100 \frac{\lim (PV)}{\lim (PV)_s - \lim (PV)_i}.$$

The absolute temperature of melting ice is obviously

$$\theta_i = 100 \frac{\lim (PV)_i}{\lim (PV)_s - \lim (PV)_i},$$

and the *centigrade thermodynamic scale* is obtained from the gas scale simply by shifting the zero point by the amount θ_i; thus

$$t = \theta - \theta_i = 100 \frac{\lim (PV) - \lim (PV)_i}{\lim (PV)_s - \lim (PV)_i}.$$

The Kelvin temperature scale

The absolute gas scale is defined for a range of temperatures in which a substance remains in the gaseous phase. The lowest temperature, therefore, that has a meaning on this scale is that at which helium liquefies. We cannot infer the existence of an absolute zero of temperature by means of a scale based upon the properties of gases. In order to establish a temperature scale that is significant at all temperatures, we must avoid considerations which depend on the behavior of the *coordinates* of a particular system. We search, instead, for a physical property which is defined (that is, measurable) for all states of matter.

The heat absorbed or rejected during a reversible isothermal process is such a quantity. Since this amount of heat depends on the extent of the process, it is necessary to find a way of limiting the various isothermal processes used. This is done by taking those portions of isotherms bounded by the same reversible adiabatics. If Q_1 and Q_2 represent the heats transferred in these restricted portions of isotherms, then it follows from the first and second laws of thermodynamics that the ratio Q_1/Q_2 is independent of (1) the nature of the system, (2) the phase in which a system may exist, and (3) the choice of the two bounding adiabatics. It may then be shown that

$$Q_1/Q_2 = \psi(t_1)/\psi(t_2),$$

where $\psi(t)$ is some function of the temperature as measured on any convenient scale.

The *Kelvin scale* is now defined by the equation

$$Q_1/Q_2 = T_1/T_2.$$

To define one Kelvin temperature, it is necessary to provide further that the temperature interval between the standard fixed points is

$$T_s - T_i = 100,$$

whence

$$T = 100\, Q/(Q_s - Q_i).$$

It is a simple matter to show that, within the region in which θ is defined, $\theta = T$.

Concluding remarks

The line of reasoning followed in this paper does not require any explicit use of, or reference to, the so-called "equation of state" of a system. This is as it should be, since the temperature concept transcends the specific peculiarities of individual systems and expresses a macroscopic property common to all macroscopic systems.

It should be pointed out that the temperature concept has been established here for that branch of physics more properly called *thermostatics*. Since the procedure adopted is an operational one, additional comment would be necessary in order to clarify the use of the temperature concept for macroscopic systems that are not in states of thermal equilibrium. This would involve discussion of the broader problem of the relation of thermostatic concepts to macroscopic systems that either are in a steady state or are varying with the time.

The Magnetic Electron Lens

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THE growing importance of electron optics in contemporary physics makes it desirable to present some of its principal features in as simple and direct a way as possible. Electron microscopes almost always use "short" magnetic lenses, by which is meant merely a circular coil of wire so placed that it is coaxial with the electron beam. (Frequently the coil is surrounded by a soft iron shield except on the inner surface facing the axis.) When a suitable current is passed through such a coil the paths of electrons diverging from an object-point on one side can be made to converge to an image-point on the other side. Magnetic lenses are also frequently used in electron guns. The standard method of deriving the focal length of such a magnetic lens is to start with the vector potential. In the present paper the formula for the focal length is obtained without assuming a knowledge of the properties of the vector potential. The result is obtained very simply by applying Ampère's formula for the force exerted on an electron when moving in a magnetic field.

I

A "short" magnetic lens is one in which the changing of the direction of motion of the electron is effected within a distance which is small compared with the focal length of the lens. It is analogous to a "thin" lens in geometrical optics. The effect of a short magnetic lens on the path of an electron moving parallel to the axis of the coil is shown in Fig. 1. Only that part of the magnetic field which is of interest to us in this discussion is represented. The boundaries of the lens may be represented by the ill-defined regions a and b. (In contrast, a glass lens has sharply defined boundaries.) To obtain a value for the focal length of the magnetic lens, it will be sufficient, precisely as in geometrical optics, to study the effect of the lens on the path of an electron approaching along a line such as OA parallel to the axis. As the electron passes through the lens the direction of the path will be altered, causing it to be directed towards the axis. However, just as in the case of the "thin" glass lens, it will be assumed that no appreciable sideways *displacement* of the path toward the axis will take place within the region between a and b, even though there is definite *deviation* of the path.

II

It is first necessary to establish a relationship between B_r, the radial magnetic induction, and B_z, the gradient of the magnetic induction along the axis. Draw a small imaginary cylinder coaxial with the axis of the magnetic field (Fig. 2). By Gauss' theorem we have

$$B_r \cdot 2\pi r \delta l + \delta l (\mathrm{d}B_z/\mathrm{d}z)\pi r^2 = 0,$$

from which

$$B_r = -\tfrac{1}{2} r \mathrm{d}B_z/\mathrm{d}z. \qquad (1)$$

We shall write $B_r' = -B_r$, where B_r' is the actual direction of the radial field at A in Fig. 1.

III

electron moves through A (Fig. 1) with a v ity u parallel to the axis, it will experience a echanical force F_1 because of the *radial* co ponent of the field B_r'. (The axial component of the field, B_z, has no effect on the motion of an electron moving parallel to it.) The radial component of the field at A (Fig. 1) is directed in toward the axis. By Ampère's rule, F_1 is at right angles to both B_r' and the velocity u, and is directed *out* of the plane of the paper at A. Its magnitude is given by $F_1 = B_r' eu$. This force produces in the same direction as itself an

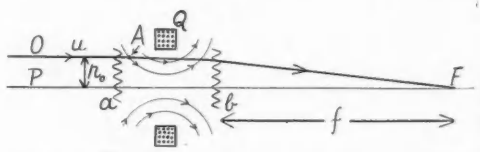

FIG. 1. "Short" magnetic lens: PF, axis of the system: Q, current-carrying coil; a and b, limits (ill defined) of the magnetic field.

acceleration[1] a of magnitude $r\ddot{\varphi}$, where φ is the angular coordinate of A with reference to the axis PF. Hence

$$ma = mr\ddot{\varphi} = B_r' eu$$

or

$$\mathrm{d}\dot{\varphi} = (B_r'/r)(e/m)u\mathrm{d}t.$$

We write $u\mathrm{d}t = \mathrm{d}z$, where $\mathrm{d}z$ is the distance parallel to the axis moved through by the electron in time $\mathrm{d}t$, substitute for B_r' by means of Eq. (1) and obtain

$$\mathrm{d}\dot{\varphi} = (e/2m)(\mathrm{d}B_z/\mathrm{d}z)\mathrm{d}z = (e/2m)\mathrm{d}B_z.$$

The total angular velocity, accumulated while the electron is going from a place where the magnetic induction is zero to where it is B_z, is obtained by integration, and is

$$\dot{\varphi} = (e/2m)B_z. \tag{2}$$

The electron has therefore acquired a linear velocity $r\dot{\varphi}$, which is directed *out* of the plane of the paper and at right angles to the line joining A to the axis. Since this newly acquired velocity is at right angles to B_z, the axial component of the magnetic field, a mechanical force of magnitude $B_z er\dot{\varphi}$ will act on the electron in such a direction as to drive it perpendicularly *toward* the axis. If F_2 is positive for the conventional direction along the axis outward, we have

$$F_2 = -B_z er\dot{\varphi}. \tag{3}$$

The resulting acceleration, outward along the radial direction, is $a = \ddot{r} - r\dot{\varphi}^2$. Hence

$$m(\ddot{r} - r\dot{\varphi}^2) = -B_z er\dot{\varphi},$$
$$\ddot{r} = -B_z(e/m)r\dot{\varphi} + r\dot{\varphi}^2.$$

Substituting for $\dot{\varphi}$ from Eq. (2), we obtain

$$\ddot{r} = -(\tfrac{1}{4})(B_z e/m)^2 r. \tag{4}$$

As the lens is "short," we shall assume (as in the case of a "thin" glass lens) that there is no appreciable *displacement* of the electron toward the axis, even though there is a finite *deviation* of its path while it is passing through the lens. We therefore write $r = r_0$, a constant. Since $(\mathrm{d}/\mathrm{d}t)(\mathrm{d}r/\mathrm{d}t) = (u\mathrm{d}/\mathrm{d}z)(u\mathrm{d}r/\mathrm{d}z)$, Eq. (4) becomes

$$\mathrm{d}r/\mathrm{d}z = -(r/4)(e/mu)^2 \int_a^b B_z^2 \mathrm{d}z. \tag{5}$$

[1] In the general case the acceleration produced by F_1 would be $r\ddot{\varphi} + 2\dot{r}\dot{\varphi}$, but as we have assumed that r does not change appreciably while the electron is passing through the lens, the second term may be neglected.

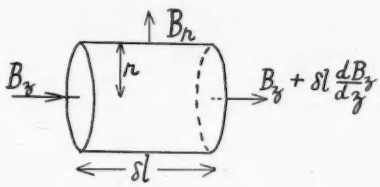

FIG. 2. To establish the relation between B_r and $\mathrm{d}B_z/\mathrm{d}z$ by the Gauss theorem.

The limits of integration bound the region over which B_z has a finite value (Fig. 1). (One could equally well use ∞ and $-\infty$, since outside a and b, B_z is zero.) The slope of the path as it emerges from the lens at b, beyond which it continues as a straight line, is $-\mathrm{d}r/\mathrm{d}z$. From Fig. 1, it is readily seen that the slope of the line is r_0/f and so $r_0/f = -\mathrm{d}r/\mathrm{d}z$. Substituting in Eq. (5), we have for the focal length of a "short" magnetic lens,

$$1/f = (e/2mu)^2 \int_a^b B_z^2 \mathrm{d}z$$
$$= (0.022/V) \int_a^b B_z^2 \mathrm{d}z, \tag{6}$$

if the energy V of the electron is expressed in electron volts and B is in gauss. Since the integrand is the square of B_z, it follows that f is always positive. This means that a magnetic lens is always a converging lens, regardless of the direction of the current. That this result is correct may be verified by applying the foregoing arguments to the case in which the direction of the lines of force in Fig. 1 is reversed.

As in the case of a converging glass lens giving a real image of an object, the image given by a magnetic lens is inverted. It has, however, an additional characteristic not found with the corresponding optical lens, namely, that the image is more or less rotated. How this comes about may be seen from Fig. 1. Consider an imaginary cylinder of radius r_0 coaxial with the axis of the magnetic lens so that the electron path lies along a generator of the cylinder. Before the electron arrives at a (Fig. 1) it has no velocity around the circumference of the cylinder. When it enters the electron lens at a, it acquires a velocity around the circumference given by $r\dot{\varphi}$. This is indicated in Fig. 3(α). Equation (2) tells us that this velocity reaches

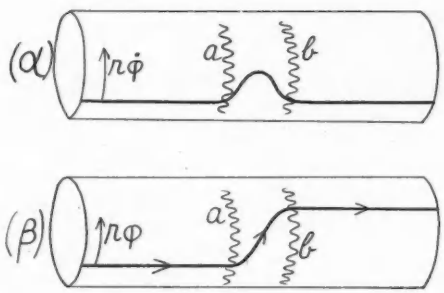

FIG. 3. (α) Illustrating the *velocity* acquired by the electron around the surface of the cylinder as a result of the force $F_1 = B_r' eu$. (β) Illustrating the resulting *displacement* of the path on the surface of the cylinder. To the right of b, the electron path has also a component pulling it inward so that it intersects the axis at the focal point. (The diagram is for the case in which the magnetic field along the axis is from right to left.)

a maximum value as the electron passes through the center of the lens where B_z is a maximum and then dies away to zero as the electron passes out beyond b. Since the velocity does not at any time reverse sign, the result will be a finite displacement of the path of the electron around the cylinder as shown in Fig. 3(β). The direction of the displacement depends on B_z and consequently on the direction of the current in the coil.

To show that the amount of rotation of the image diminishes as the "thickness" of the lens decreases, we proceed as follows. Equation (2), combined with $dz = u dt$, yields on integration an expression for the angular rotation, namely,

$$\varphi = (eu/2m)\int_a^b B_z dz = (eu/2m)\langle B_z\rangle_{Av}(b-a), \tag{7}$$

where $\langle B_z\rangle_{Av}$ is the mean value of B_z over $(b-a)$, the "thickness" of the lens. Likewise Eq. (6) can be replaced by

$$1/f = (e/2mu)^2\langle B_z^2\rangle_{Av}(b-a), \tag{8}$$

where $\langle B_z^2\rangle_{Av}$ is the mean value of B_z^2 over $(b-a)$. Combining Eqs. (7) and (8) we get

$$\varphi^2 f = u^4[\langle B_z^2\rangle_{Av}/\langle B_z\rangle_{Av}^2](b-a).$$

This tells us that if we reduce the dimensions of the lens x-fold and increase $\langle B_z^2\rangle_{Av}$ x-fold to keep f constant, then φ^2, the square of the angular rotation, will diminish with the thickness $(b-a)$. (The ratio $\langle B_z^2\rangle_{Av}/\langle B_z\rangle_{Av}^2$ will, of course, be constant if the fields are similar in every way except for scale.)

At the other extreme from that of an ideal "short" magnetic lens, we have a magnetic lens in which the field is constant over the whole distance between the object- and image-points. Such a field could, for example, be produced by a sufficiently long solenoid. Consider an electron that starts from an object point O [Fig. 4, (a) and (b)] with an axial velocity u_z parallel to the magnetic field and a radial velocity u_r at right angles to it. Insofar as the axial velocity component u_z is concerned, there will be no mechanical force on the electron, for u_z is parallel to the magnetic induction B. Since u_r is at right angles to the magnetic field, a mechanical force accelerating the electron at right angles to both u_r and B will result. The acceleration produced by this force is given by

$$ma = mu_r^2/R = Beu_r, \tag{9}$$

where R is the radius of curvature of the resulting circular path projected on a plane at right angles to the direction of the field [Fig. 4(a)]. From Eq. (9) we obtain $R = u_r/B(e/m)$. The time required to go around the circle is $T = 2\pi R/u_r = 2\pi/B(e/m)$, which, we observe, is independent of the radial velocity component u_r. In time T, the electron will travel a distance $L = Tu_z = 2\pi u_z/B(e/m)$ in the direction of the field. As a consequence of the fact that T is independent of u_r, every electron starting out from O with an axial component u_z will reach I at the same time, irrespective of the value of its radial component u_r. This statement is true as it stands but it does not correspond to what one generally finds

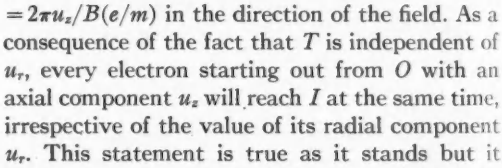

FIG. 4. (a) Showing the projection of the path on a plane at right angles to the magnetic field. (b) Showing the path on a plane which rotates about OI with constant speed so that the plane rotates through π and the vector u_r through 2π, while the axial velocity component takes the electron from O to I. (c) Illustrating various paths projected on a plane normal to the magnetic field for various values and initial directions of u_r.

in an experiment. One may easily obtain experimentally a situation in which all the electrons start out from O with the same *total* velocity u. The foregoing consideration now holds only so long as the radial velocity component u_r is small enough to insure that the axial component u_z will be substantially identical with u, and so can be treated as a constant. Thus, subject to the restriction that $u_r \ll u$, all electrons starting out from a point such as O will arrive at I, a point which is on the same line of force as O. Other points in the same plane as O will give rise to image-points in the same plane as I. These planes are at right angles to the direction of the field. The image will be erect and of the same size as the object.

It is to be observed that, in constructing an image from an object, one does not use the method customary in geometrical optics and with "short" magnetic lenses. There is no lens "center" through which all rays pass undeviated from an object-point to the corresponding image-point, nor is there a focal point through which pass all rays that were parallel to the axis on the other side of the lens.

Since the rotation of the image produced by a "short" magnetic lens increases with the thickness of the lens, it is at first sight surprising to find that this uniform field of unlimited length produces an erect image, that is, one in which there is apparently no rotation at all. A consideration of Fig. 4(*a*) or 4(*c*), however, indicates that the rotation of the image is in reality a rotation through 360°, which is, of course, indistinguishable from no rotation at all.

On the Many-Valued Logics

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"I INTEND to develop the idea that the old logic was devised for a world that was thought to have hard outlines, and that, now that the new mechanics has shown that the outlines are not hard, the method of reasoning must be changed."[1]

So said Professor C. G. Darwin. He suggested that the new logic might arrive via statistical theory. He is only one of many physicists who have believed that the new developments in physics would compel new ways of thinking. The suggestions regarding the form which the new logic might take have been numerous and very vague; anyone with a concrete idea would have acted on it instead of merely suggesting it. Nevertheless, with so many people talking about a new logic, something should be done about it.

What?

Perhaps a good way to start would be to list the requirements that a logic must satisfy, and then see what possibilities are left open.

If physics is to consist of anything beyond a mere compilation of data, one must have physical theories. And what is a physical theory? Merely this: After one discovers by experiment that a certain physical structure exists, he exhibits a mathematical structure which approximates the physical structure. If the mathematical structure approximates the physical structure closely, the theory is called good. If a study of the mathematical structure reveals new and unsuspected details of the physical structure, the theory is an outstanding success. Examples of outstanding successes are Newton's theory of gravitation, Maxwell's theory of electromagnetic radiation, Einstein's theory of general relativity, the quantum mechanics, and so forth.

So the first requirement on our new logic is that it be adequate for the formation of physical theories. Let us be more precise. The new logic, together with its resultant mathematics, must be sufficiently rich in diverse but interrelated and interlocking parts so that mathematical structures will be available to approximate physical structures of any desired degree of complexity.

[1] Darwin, "Logic and probability in physics," Phil. of Sci. **6**, 48–64 (1939); presidential address to the Mathematics-Physical Section of the British Association for the Advancement of Science.

To put it briefly, the new logic must have a wealth of structure.

This appears to be the only essential requirement on the new logic!

However, physical theories of the familiar sort make their contact with reality through measurement. Presumably a new logic might make contact with reality some other way (thus avoiding the principle of indeterminacy, perhaps). Since I cannot give even a vague suggestion for anything so bizarre as this, I will bow to convention to the extent of ruling that the new logic must make contact with reality through the medium of measurement; that is, the new logic must contain those indispensable adjuncts of measurement—numbers.

With the requirements on the new logic reduced to such a drastic minimum, it certainly seems plausible that such new logics must exist—indeed, a great many of them. But how can they be found? One cannot just sit down and say, "Now I will think of a new logic; one with numbers and a wealth of structure."

Past experience with thinking up new ideas suggests that one way of proceeding would be to list the presuppositions of the old logic, then deny one of them and try to see what one gets. I shall discuss a result of one such attempt.

Before describing the new logic, I wish to estimate its importance. With (probably) a great many new logics to be discovered, it is quite unlikely that the first to be discovered should turn out to be the one which will unify twentieth-century physics. Nevertheless, an attempt to unify modern physics by means of the new logic will require a scrutiny of familiar situations from a novel point of view, and this cannot fail to bring to light useful ideas that are at present unnoticed. Also the effort to acquire the new point of view will free our minds of preconceived ideas and facilitate the discovery of other new points of view. So the new logic should stimulate invention, and the chance still remains that it will turn out to be useful in its own right.

Now for the new logic. In the old logic, a basic principle is that any statement is either true or false. One would expect that to deny this would leave us in an intolerable mess, but it does not seem to.

First, if there can be another possibility for a statement besides the usual possibilities of true or false, there might as well be several. So we classify logics according to the total number of possibilities for a statement. Logicians refer to the possibilities as *truth values*. In the old logic there are two possible truth values, namely, true and false; so the old logic is a two-valued logic. When we admit one other possibility for a statement besides true and false, we have a three-valued logic. When we admit two other possibilities for a statement besides true and false, we have a four-valued logic. And so on. There seems to be no reason for not allowing an infinite-valued logic, but I do not intend to cudgel my brain with that monstrosity for a while yet.

Before proceeding further, we will point to a possible use for a three-valued logic in modern physics. In Dirac's interpretation of his "principle of superposition," he affirms that between the states of an atomic system there exists a peculiar relationship such "that whenever the system is definitely in one state, we can equally well consider it as being partly in each of two or more other states. The original state must be regarded as the result of a kind of *superposition* of the two or more new states in a way that cannot be conceived on classical ideas."[2] In a two-valued logic, a system is either in a state or not in it. In a three-valued logic, a system can fail to be either in a state or not in it, which situation can be described by saying that the system is partly in the state.

Returning to the description of our new logic, we recall that one requirement is that it contain numbers. So we try to go about getting numbers. In a two-valued logic, one can derive all of modern mathematics (which certainly includes numbers) from only four basic ideas.[3] These are described as follows:

First, the idea *and*. If P and Q are two statements, one can form a new statement by combining them with an *and*. We will denote the new statement by $P \& Q$. It is standard practice

[2] See P. A. M. Dirac, *The principles of quantum mechanics* (ed. 1), p. 8.

[3] See W. V. Quine, *Mathematical logic* (Norton). Strictly speaking, Quine uses only three ideas. However, his three ideas can be built up out of our four basic ideas, and our four ideas are rather more simple than Quine's three, so for expository purposes I have chosen the four.

to refer to P & Q as the "logical" product of P and Q.

Second, the idea *not*. If P is a statement, the contradictory statement to P can be formed by inserting a *not* in P at the grammatically correct place. We will denote the contradictory of P by

FIG. 1.

	T	F
T	T	F
F	F	F

$\sim P$. Thus if P is "It is raining," $\sim P$ would be "It is not raining."

Third, the idea *for all*. If P is a statement involving a variable x, one can form the new statement, "P for all x." We will denote the new statement by $(x)P$. Thus if P is "$\sin^2 x + \cos^2 x = 1$," then $(x)P$ is "$\sin^2 x + \cos^2 x = 1$ for all x." To realize that there is a distinction between P and $(x)P$, let P be "$x^2 - 4x + 3 = 0$." Then $(x)P$ is "$x^2 - 4x + 3 = 0$ for all x." In this case $(x)P$ is irrevocably false, whereas P can be true (namely, when $x = 1$ or 3).

Fourth, the idea *is a member of*. If α is a collection of objects and x is an object, one can form the statement that the given object x is a member of the given collection α. We denote this statement by "$x \epsilon \alpha$."

Now since one can get numbers from these four ideas in a two-valued logic, we shall take pains to get these four ideas into our new logic with the hope that they will produce numbers there also. To simplify matters for the reader, we shall restrict the discussion to a three-valued logic. We denote our three truth values by T, *?* and F. We think of T as signifying true, F, false, and *?*, the new possibility which we are allowing.

First consider *and*. Let us pause to remind ourselves of the behavior of *and* in the old two-valued logic. Here, the possibilities for P, Q and P & Q are either T or F and no other. Also P & Q is T if and only if both P and Q are T. (The reader can see this at once by reference to the meaning of the word *and*.) This can be exhibited in a "multiplication table" for *and* (Fig. 1). It is to be read as follows. Across the top line are written the possible values of P. Down the left-hand column are written the possible values of Q. Now, knowing the value of P and the value of Q, one can find the value of P & Q by looking in the table under the value of P and to the right of the value of Q. It is easily seen that the table agrees with the condition that P & Q is T if and only if both P and Q are T. Also this condition could not be fulfilled for any other multiplication table. For, by the condition, a T must go in the upper left-hand corner where both P and Q are T, and cannot go elsewhere, since either P or Q fails to be T at every other place. So the remaining places must be filled with F's.

Now proceed to a three-valued logic. In a two-valued logic, the condition "P & Q is T if and only if P and Q are both T" described the idea *and* completely, so we take this to be our definition of *and* in a three-valued logic. Let us form the multiplication table for P & Q in three-valued logic. Write the possible values of P across the top line, and the possible values of Q down the left-hand column (Fig. 2). By our

FIG. 2.

	T	?	F
T	T		
?			
F			

condition, a T must go in the upper left-hand corner where both P and Q are T. Also no other square can contain a T. But, for all we have said, either F or *?* can go in any of the other squares. In other words there are many ways of filling out the other squares, depending on whether we put F's in all, *?*'s in all, or F's in some and *?*'s in the rest. With eight squares to be filled, and two possibilities to each square, there are altogether 2^8 or 256 possible ways of filling the squares. Moreover, each method of filling the squares satisfies our condition and so is a kind of *and*. In other words, there turn out to be 256 different kinds of *and* in a three-valued logic!

Apparently the only thing that can be done about the matter is to pick out the *and* that one likes best, and try to ignore the rest. Judging from the experience thus far available, this is feasible.

Not everybody likes the same *and*. Post, Łukasiewicz, and Rosser[4] prefer the *and* with the table in Fig. 3. This *and* is got by agreeing that *?* is falser than T, and F is falser than *?*, and that $P \,\&\, Q$ should be exactly as true as the falser of its two factors. With such an *and*, one does not commit himself as to what *?* denotes. Bočvar[4] chooses to commit himself as to what *?* denotes, and thinks of it as meaning "undecidable." If either P or Q is undecidable, Bočvar feels that $P \,\&\, Q$ is consequently undecidable (not everyone agrees to this). So Bočvar favors an *and* with the table in Fig. 4.

The situation with regard to *not* is very similar. When P is T, $\sim P$ must fail to be T, and when P is F, $\sim P$ must fail to be F. Whereas that determines the behavior of *not* completely for a two-valued logic, it allows considerable leeway for a three-valued logic. Among the twelve possible *nots* in three-valued logic, three have received special study. They are indicated in the three tables in Fig. 5, in which the possible values of P are written under P, and the corresponding values of $\sim P$ are written under $\sim P$. The table on the left was favored by Post, the middle one by Łukasiewicz, Bočvar, and Rosser, and the one on the right by Bočvar (who seemed to need two *nots*). The question about *not* on which there was disagreement, involved the status of $\sim\sim P$. In two-valued logic, a double negative yields a positive. This can be stated two ways (which are the same for two-valued logic, but not for three-valued logic). One way of saying that two negatives make a positive would be to say that $\sim\sim P$ is the same as P. In a three-valued logic, this could be the case only for the middle one of the three *nots* tabulated in Fig. 5. Another way of saying that two negatives make a positive would be to say that $\sim\sim P$ is T when

	T	?	F
T	T	?	F
?	?	?	F
F	F	F	F

FIG. 3.

	T	?	F
T	T	?	F
?	?	?	?
F	F	?	F

FIG. 4.

P	$\sim P$
T	?
?	F
F	T

P	$\sim P$
T	F
?	?
F	T

P	$\sim P$
T	F
?	T
F	T

FIG. 5.

and only when P is T. This would be the case for either the middle or right-hand *not* shown in Fig. 5. That still leaves Post's *not* (the one on the left) to be explained. Post's feeling in the matter was that if a double negative yields a positive in two-valued logic, then in three-valued logic, a triple negative should yield a positive. His *not* is chosen accordingly.

We shall not attempt to give any details of the behavior of *for all* in three-valued logic, except to say that it is rather like *and*, only worse.

By comparison, the situation with regard to $x \epsilon \alpha$ (the reader is reminded that this means "x is a member of the collection α") is quite simple. When x is in the collection α, we assign the value T to $x \epsilon \alpha$, and when x is not in the collection α, we assign the value F to $x \epsilon \alpha$, and in all other cases we assign the value *?* to $x \epsilon \alpha$. It must be remembered that we are dealing with a three-valued logic, and so must curb any feeling (resulting from two-valued habits of thought) that there would not be any case where $x \epsilon \alpha$ is *?*.

We shall digress again, this time to reassure the reader on a delicate point. In describing the *and*, we used *and*, and in describing the *not*, we used *not*. Perhaps this is circular. The situation when we are describing the two-valued *and* is quite like that of the person writing a book on English grammar and writing it in English. To describe his English grammar, he must use English sentences, and these embody English grammatical structure. A person without knowledge of English grammar could not acquire a knowledge of English grammar from the book because his ignorance of English grammar would prevent him from comprehending the book. Similarly, when we describe the two-valued *and*, we must use *and* (or some equivalent such as *both*); this second *and* is necessarily the *and* of two-valued logic. So if one did not know the meaning of *and* to start with, he could not learn it from our explanation. However, if one does

[4] See the "Historical and Bibliographical Remarks" at the end of this paper.

know the meaning of *and*, he must agree that our explanation is correct.

However, everybody already knows the meaning of the two-valued *and*, so it is perfectly all right if our explanation of the two-valued *and* is not enlightening. What is much more to the point is whether the explanation of the three-valued *and* is enlightening. Here the situation is quite different. We use the two-valued *and* to describe the three-valued *and*. So one does not need to know the meaning of the three-valued *and* in order to comprehend the description; consequently he can learn the meaning of the three-valued *and* from the description. The situation is perfectly analogous to that of a book on Latin grammar written in English. A person with absolutely no knowledge of Latin grammar could learn Latin grammar from such a book, whereas he would be helpless if given a book on Latin grammar written in Latin.

Thus we proceed to clear up the question of the apparent circularity of the two *ands*. When one *and* is two-valued and the other *and* is three-valued, the two *ands* actually are totally distinct ideas, and no circularity exists in using one to describe the other. Incidentally we have answered certain critics who claim that an explanation of three-valued *ands* should be made in terms of three-valued *ands*. On the contrary. An explanation of three-valued *and*s must be made in terms of whatever kind of *ands* the reader already understands.

This brings up a horrible thought. If we rear a generation to think in terms of three-valued logic, will it some day become necessary to write an explanation in terms of three-valued *ands* of the meaning of the two-valued *and*?

One matter connected with "$x \epsilon \alpha$" is of especial interest. For most collections of objects, the collection itself is not a member of the collection.[5] That is, for most collections α, it is not true that $\alpha \epsilon \alpha$; instead, $\sim(\alpha \epsilon \alpha)$. Now define a collection U to consist of those collections α for which $\sim(\alpha \epsilon \alpha)$; that is, U is the collection of exactly those collections which are not members of themselves. Is U a member of itself; that is, is $U \epsilon U$? In two-valued logic only two cases can arise: either it is or else it is not; that is, $U \epsilon U$ is either T or F.

Case 1. Suppose that $U \epsilon U$ is T; that is, U is a member of U. However, by definition, each member of U is not a member of itself. So U (being a member of U) is not a member of itself, namely, of U. This contradicts our assumption that $U \epsilon U$ is T. So that possibility is out.

Case 2. Suppose that $U \epsilon U$ is F; that is, U is not a member of U. Thus U is not a member of itself. However, by definition, every collection that is not a member of itself is a member of U. So U is a member of U. This contradicts our assumption that $U \epsilon U$ is F. Consequently, that possibility is out.

At this point a third possibility would be very helpful. If one could simply classify $U \epsilon U$ as ?, the situation would be saved.

Of course, the entire discussion involving U is open to a variety of objections. Some extremists, who are determined to preserve a two-valued logic at any cost, declare that the whole business involving U is senseless. However, $x \epsilon \alpha$ is necessary in mathematics, and is used in mathematics in the way we used it in discussing U. If our discussion of U is senseless, so is mathematics senseless. "Very well," say the extremists, "so mathematics is senseless."

These are brave words, and show an admirable unwillingness to be bound by tradition, but they ignore certain profound realities. In particular, they ignore the fact that engineering and physics and chemistry use mathematics in a basic manner. And the engineers and physicists and chemists produce results which the extremists eat and wear and ride in and listen to, and which cannot be ignored very conveniently.

Unfortunately the complications arising in our discussion of U must be reckoned with, and the use of three-valued logic seems to be one way of doing it. Not that everything is a bed of roses even with a three-valued logic. The definition of U involves a *not*, and in three-valued logic the use of *not* involves further complications. However Bočvar has shown that one can apparently clear up the U business by means of three-valued logic.

Let us see if the three-valued logic is living up to the requirements that we laid down earlier. We asked for wealth of structure. Well, we are

[5] For a collection which is, take the collection whose members consist of absolutely everything, and so include the collection itself along with everything else.

getting it in abundance. In fact, after working a while with n-valued logics, one begins to wonder if there is such a thing as too much structure.

We also asked for numbers. We put *and*, *not*, *for all*, and $x\epsilon\alpha$ into our three-valued logic with the hope of getting numbers. It turns out that we get numbers. We will not indicate how the numbers arise, but will merely remark that because of the multiplicity of *ands*, *nots*, and so forth, we get not one set of numbers but many sets. There is a set that behaves like two-valued numbers as long as one concentrates on the truth values T and $?$, and another set that behaves like two-valued numbers as long as one concentrates on the truth values T and F, and still other sets that appear to flutter in random fashion from one truth value to another. For instance, for some sets, $2+2=4$ is T, and for other sets, $2+2=4$ is $?$. However, one never gets $2+2=4$ to be F, even if one picks the 2's and the 4 from three different sets. Lest the reader be too outraged at having $2+2=4$ be anything but T, let me point out that with several hundred 4's, if one picks the same pair of 2's every time, he would not want $2+2=4$ to be T every time. Add to this the fact that there are several hundred 2's also, and that the two 2's do not even have to be the same ones, and the wonder is that $2+2=4$ never gets to be F.

This may look too confused to be of any possible use, but once one gets over the shock, certain signs of order appear. For instance, it is disconcerting to use two different 4's and get $4=4$ to be $?$ instead of T, but at least $4=4$ is never F. Also $3=4$ is never anything but F, no matter what 3 or 4 you pick. So the numbers stay in order, as they should. They merely have some sidewise motion, so to speak. And if one is not prone to seasickness, there may be situations where such sidewise motion is very helpful. In fact, one cannot help feeling that something of the sort would be handy in modern physics. The physicist introduces wave functions and all sorts of complications in order to achieve compatibility between a mathematics with sharp edges and a physics without sharp edges. The situation cries for a mathematics without sharp edges, and a three-valued logic happens to produce a mathematics without sharp edges. We do not know enough about it yet to judge whether or not it is the right one, but at least it encourages us to believe that the right one exists, and to look for it.

Historical and Bibliographical Remarks

An unwillingness to accept the principle that every statement is either true or false is apparently nothing new. It is said that the philosopher Hegel challenged this principle. Some philosophers divide statements, not into true and false, but into necessary (=necessarily true), impossible (=necessarily false), possible but not necessary, and so forth. In 1896, Hugh MacColl published a 28-page paper [Proc. London Math. Soc. **28**, 156–183 (1896)] giving a quite detailed study of a logic with the three values, "necessary," "impossible," and "possible, but not necessary." As examples of these three values, he exhibited the three statements "$2=2$," "$3=2$," and "$x=2$."

However, these earlier writers only repudiated two-valued logic to a very slight extent. For a thoroughgoing repudiation of the two-valued logic, such as indicated in this paper, one must turn to much more recent writers. Jan Łukasiewicz has published several papers (the first in Polish, in 1920) which are often looked upon as the real beginning of the subject of many-valued logics. A satisfactory exposition of the work of Łukasiewicz (and his associate, Tarski) is to be found in Lewis and Langford, *Symbolic logic* (Century, 1932), pp. 213–234. Another important contributor was E. L. Post, whose paper, "Introduction to a general theory of elementary propositions" [Am. J. Math. **43**, 163–185 (1921)] is a landmark in the subject. Two recent papers which mark advances over the work of Łukasiewicz and Post are:

D. A. Bočvar, "On a three-valued calculus and its application to the analysis of contradictions," Matématičéskij sbornik, n. s. **4**, 287–308, (1939). This paper is in Russian, but a review in English, and a correction to this review, appears in J. Sym. Logic **4**, 98–99 (1939); **5**, 119 (1940).

B. Rosser, "The introduction of quantification into a three-valued logic," preprinted for the members of the Fifth International Congress for the Unity of Science, Cambridge, Mass., 1939, as from J. Unified Sci. (Erkenntnis), vol. 9. This volume is not yet available in this country, due to dislocations in Europe. However, the article is covered by the reprint service of *Mathematical Reviews* (see cover 2 of that journal; order by title; length of paper, 6 pages).

The development of numbers in a many-valued logic has not yet appeared in print, but will be incorporated in an article by the present author in a forthcoming issue of the *Journal of Symbolic Logic*. Since this paper will also feature certain technical improvements, the development will be started from the very beginning; hence the new paper will be independent of the author's earlier paper, and will supersede it completely.

For anyone desiring a complete bibliography of many-valued logics, the bibliographical material in the *Journal of Symbolic Logic* is unexcelled; see vol. 3, p. 198 (1938) and vol. 5, p. 181 (1940), under "Many-valued logic."

Loudness and Intensity

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NOT more than thirty years ago, physicists attempted to make little, if any, distinction between the concepts of *loudness* and *intensity*. The earliest, most useful, quantitative relationship established between those two concepts is summarized in the Weber-Fechner psychological law[1] which states, in Fechner's words, that "the sensation increases as the logarithm of the stimulus." This law implies, for example, that the ratios of stimuli in a stepped scale of least audible increments are equal to one another in sensation. If this were true, 40 least audible increments would produce twice as much sensation as 20 such increments. Less than twenty years ago, many loudness meters, manufactured under various names, were calibrated on a logarithmic scale and were used to measure loudness levels. By 1928, those interested in acoustic correction in rooms suspected that the logarithmic relationship was not exact. The many experiments performed since 1930 have shown conclusively that loudness is only approximately proportional to the logarithm of the intensity of sound and that the departure in linear relationship is considerable for wide intervals in intensity levels.

Because of the rapidity with which new relationships have been established between loudness concepts and intensity,[2] many of the definitions appearing in our recent college textbooks are faulty. Thirty general physics textbooks, written by different authors, and published since 1928, were examined. Four of these textbooks published prior to 1938 did not use the term *loudness*. Nine authors state that loudness depends on the square of the amplitude, an assumption usually based on the idea that loudness depends upon intensity in some simple way; in fact, a few of the currently used physics textbooks make no essential distinction between *loudness* and *intensity*. Ten authors define loudness in terms of sensation and end with the idea that loudness is proportional to the logarithm of the stimulus. The Weber-Fechner law is occasionally mentioned to justify the mathematical statement. Two authors state that the relation between loudness and intensity is not simple and that loudness depends on pitch also; the reader is referred then to psychologists for further discussion. Four authors define loudness in accordance with the customary usage of the word by specialists in the field of acoustics. Finally, four authors, in preparing later editions, did not revise the faulty definitions and concepts used in their discussion of loudness.

Loudness[3] is defined as the magnitude of the auditory sensation produced by the sound. The measurement of loudness is based upon the mental estimates of many typical observers and is best recorded quantitatively, for the present, by the use of graphs. The two most commonly used graphical representations are (*a*) the loudness *versus* phon[4] scale[5] at 1000 cycle/sec (Fig. 1),[6] and (*b*) the equal-loudness contour scale (Fig. 2). The solid line of Fig. 1 gives a temporarily adopted relation between the change

[1] Fechner, *Elements of psychophysics* (1860).

[2] The definition adopted by the American Standards Association is: "The sound intensity of a sound field in a specified direction at a point is the sound energy transmitted per unit of time in the specified direction through a unit area normal to this direction at the point. The unit is the erg per second per square centimeter but sound intensity may also be expressed in watts per square centimeter." H. A. Frederick, J. Acous. Soc. Am. **9**, 60 (1937).

[3] The definition adopted by the American Standards Association is: "The loudness is that subjective quality of a sound which determines the magnitude of the auditory sensation produced by that sound." H. A. Frederick, reference 2.

[4] Adopted as a unit of equivalent loudness level measurement at a meeting of delegates of the International Electrotechnical Commission held in Paris, July, 1937; this committee worked under the auspices of the International Standards Association. Fletcher, Bell. Lab. Rec. **16**, 213 (1938).

[5] The solid line of Fig. 1 is taken from a paper by H. Fletcher and W. A. Munson, J. Acous. Soc. Am. **5**, 82 (1933). The dotted line is from more recent data of H. Fletcher and W. A. Munson, J. Acous. Soc. Am. **9**, 1 (1937). The solid line was adopted for noise measurements by the American Standards Association, Feb. 17, 1936 [J. Acous. Soc. Am. **8**, 143 (1936)]. See Geiger, J. Acous. Soc. Am. **11**, 308 (1940), for an extension of the data in the Fletcher and Munson table to smaller loudness differences. For an earlier tentative adoption, see also, "Proposed standards for noise measurements," J. Acous. Soc. Am. **5**, 109 (1933).

[6] The word "cycles" is often substituted for the correct physical expression, *cycle/sec*, in acoustic literature.

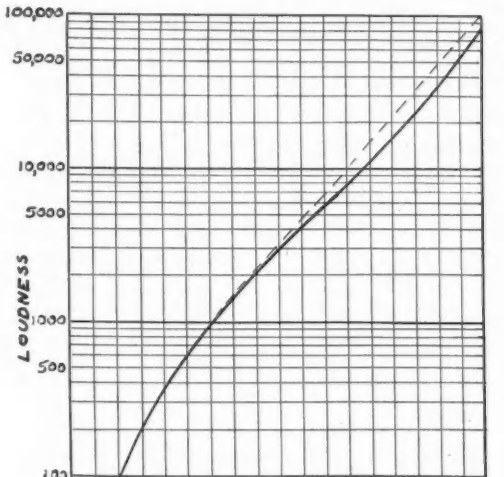

FIG. 1. This curve is for 1000 cycle/sec, the reference frequency to which all loudness measurements are referred. (See reference 5 for details.) The *phon*, as used in this curve, is the number of decibels the 1000-cycle/sec tone is above the reference threshold of 10^{-16} w/cm.[2]

in loudness for any corresponding change of intensity at stated phon levels, while Fig. 2 pictures equal-loudness-level contours[7] for all frequencies when compared with stated phon levels of the 1000-cycle/sec tone used as the reference frequency standard.

The loudness-phon curve of Fig. 1 indicates that the increase in loudness for the 1000-cycle/sec reference tone at stated phon (or energy) levels is not constant for constant increases of intensity. The ordinates are expressed in arbitrary loudness units such that doubling, for instance, any given number increases the loudness by two. The numbers are chosen experimentally so that the scale starts with a loudness number 1 for zero intensity (or loudness) level on the abscissa axis. All loudness numbers are obtained experimentally by mental judgment tests. Figure 1 then represents a true loudness scale in that doubling, trebling, and so forth, the numbers increases the loudness by that amount. The numbers on the abscissa scale are in decibels above some arbitrary level of intensity. The *phon* is the number of decibels the 1000-cycle/sec tone is above the reference threshold of 10^{-16} w/cm^2. A more general definition of the phon, as applied to loudness levels of other frequencies, will be given later.

An equation that gives the number of decibels[8] above some reference intensity or excess pressure is

$$N = 10 \log I/I_0 = 20 \log P/P_0,$$

where I/I_0 represents the ratios of two intensities and P/P_0 represents the ratios of two root-mean-square excess pressures. Since the ear is sensitive to excess pressure amplitudes rather than to velocity or displacement amplitudes, excess pressure amplitudes are convenient quantities to use in loudness measurements. In fact, intensity of a sound may be considered indefinitely defined in the usual four-walled room where standing waves prevail. To avoid room interference, loudness measurements are made in an acoustically dead room or outdoors. The initial intensity I_0 has been standardized to 10^{-16} w/cm^2 ($P_0 = 0.0002$ dyne/cm^2 rms pressure) for a free wave since the threshold of hearing at 1000 cycles/sec approximates this value.

It is important to note that the decibel is a unit of energy level and not a unit of loudness. The term *loudness level*, used in Fig. 1 to mean the same thing as the decibel level for a 1000-cycle/sec tone, has proved to be confusing to many people. A 1000-cycle/sec tone with a loudness level of 40 phons (or db) does not indicate a loudness twice that of a loudness level of 20 phons. But a 1000-cycle/sec sound of 4000 loudness units, as shown on the ordinates of Fig. 1, will have twice the loudness of a 1000-cycle/sec sound of 2000 loudness units. Similarly, a 1000-cycle/sec sound of 5000 loudness units will have five times the loudness of one of 1000 loudness units and will,

[7] Adopted by the American Standards Association, Feb. 17, 1936 [J. Acous. Soc. Am. 8, 143 (1936)]. See also reference 5 for information concerning an earlier tentative adoption.

[8] "The decibel is one-tenth of a bel, the number of decibels denoting the ratio of two amounts of power being ten times the logarithm to the base ten of this ratio. The abbreviation db is commonly used for the term decibel." H. A. Frederick, reference 2.

according to the graph, require an increase in intensity of about 20 db. It will be noted that the increase in phons needed to produce a stated multiple increase in loudness depends upon the phon level but is more nearly constant above a phon level of 40 db.

There have been many ways of arriving at experimental results[9] such as those pictured in Fig. 1. Four of the widely used methods will be described briefly. Of the four, three are comparison methods to obtain equal loudness and may be characterized as the *one-ear two-ear method*, the *two-component-tone method* and the *ten-component-tone method*.

In the one-ear two-ear method, one listens to a tone of 1000 cycle/sec sounded with equal intensity simultaneously at both ears, then, to the same 1000-cycle/sec tone at one ear increased sufficiently in intensity to be equal in loudness to the former tone that was sounded simultaneously at both ears. The two-component-tone method consists of choosing a tone far removed in frequency from the 1000-cycle/sec tone and making it equally loud to the 1000-cycle/sec tone; these two tones are then sounded together and a determination is made of the intensity of a 1000-cycle/sec tone that will sound equally loud compared to the two tones. The ten-component-tone method is similar to the two-component-tone method, except that 10 tones far removed in frequency are determined experimentally to be equally loud to a 1000-cycle/sec tone; then the 1000-cycle/sec tone is raised in intensity until it is found to be as loud as the complex tone of ten-component tones. In all component-tone methods, the frequencies must be separated sufficiently so that the nerves stimulated by the tones will not overlap. Non-overlapping of nerve stimulation is assumed for the one-ear two-ear method also.

The fourth method consists of the estimation of some definite increase or decrease in loudness by listening first to the reference tone and then to the tone of different intensity level whose estimation of change in loudness is required. For instance, one may determine the increase in intensity necessary to double the loudness, or determine the decrease in intensity necessary to halve the loudness. The experimental results by all four methods just described agree reasonably well, although certain irregularities are being investigated.

Each of the loudness contour lines of Fig. 2 shows the intensity levels of tones of different frequencies that are judged to be equal in loudness to the intensity level of the 1000-cycle/sec tone included as a point in the loudness contour line. The phon levels (or loudness levels) for the 1000-cycle/sec tone are shown for 10-db intervals. There will be then, a contour line passing through each of the 10-db increases in phon levels for the 1000-cycle/sec tone. There is a general crowding of contour levels in the low frequency region, indicating that a given change of loudness in this region will be caused by a smaller change in intensity than at higher frequencies. Thus, a change in intensity of 5 to 6 db at 60 cycles/sec will produce about the same difference in loudness as a 10-db change at 1000 cycles/sec. For a wide range of frequencies above 500 cycles/sec, approximately the same change in intensity produces a given change in loudness.

In interpreting Fig. 2, concepts of both *intensity level* and *loudness level* are needed. The *intensity level* is defined as the intensity of any frequency, or complex sound, measured in decibels above the base reference level of 10^{-16} w/cm^2 (or 0.0002 dyne/cm^2). The *loudness level* of any frequency, or complex sound, is defined as the loudness level of the equally loud 1000-cycle/sec reference tone as judged by listeners and is expressed in phons. As in Fig. 1, our definitions show that intensity levels and loudness levels of Fig. 2 have the same numerical decibel magnitudes at 1000 cycle/sec. Thus, a 1000-cycle/sec tone with an intensity level of 60 db also has a loudness level of 60 phons. For other frequencies, we note the phon level of the 1000-cycle/sec tone that is judged to be equally loud. This becomes the loudness of the sound in

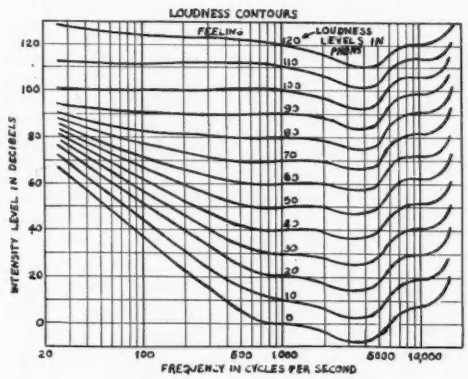

FIG. 2. Each of the 13 contour lines is drawn through intensity levels of tones of different frequencies that are judged to be equal in loudness to the 1000-cycle/sec tone under consideration. If the equally loud 1000-cycle/sec tone is N db above 10^{-16} w/cm^2, the contour line is said to have a loudness level of N phons. In general, intensity levels and phon levels do not agree numerically except at 1000 cycles/sec. [Contour lines redrawn from curves given by Fletcher and Munson.]

[9] H. Fletcher, J. Acous. Soc. Am. 9, 275–293 (1938).

question and is measured in phons. For example, an 80-cycle/sec tone has a loudness level of 30 phons (Fig. 2) when the intensity level is 60 db; a loudness level of 50 phons when the intensity level is 70 db. The *phon*, then, measures the loudness level of a tone and is the number of decibels the 1000-cycle/sec tone is found to be above the reference intensity level when judged by observers to be equal in loudness to the sound in question.

The equal-loudness contour curves of Fig. 2 may be obtained[10] by listening with both ears first to the 1000-cycle/sec reference tone and then to the frequency under examination alternately while regulating the intensity under examination until the two sounds appear to be equally loud. The 1000-cycle/sec tone is adjusted in advance to any predetermined intensity level. Actually, the curves of Fig. 2 were obtained by a different process,[11] too long to be described here.

Loudness measurements, determined in accordance with the curves of Fig. 2, are made by listening with both ears to the sound under examination and to the 1000-cycle/sec reference tone alternately, adjusting the intensity of the reference tone until they both appear to be equally loud. When judged to be equally loud, the intensity level (or loudness level) of the 1000-cycle/sec tone, read as N db on the attenuator, may be called N phons, and represents the loudness level of the sound under test. The intensity level of the sound under examination, if desired, will be measured in decibels. Generally speaking, the intensity level will not be the same as the phon level. Thus, the intensity levels for most low frequencies are higher than their loudness levels.

Except for the desirability of distinguishing the intensity level of the 1000-cycle/sec tone as a reference frequency and, therefore, as a reference loudness standard, there is no value in expressing intensity levels of sounds in any other unit than the decibel.

Two other units commonly used, especially by psychophysiological workers in acoustics, to express the magnitude of sensation of a sound are the *sensation level* and the *sone*.[12] The *sensation level* is commonly defined as the number of decibels that sound is above normal threshold reference intensity. The threshold of hearing is usually more convenient as a reference point for making loudness measurements than measuring the absolute sound intensity. Conversions to graphs such as Fig. 1 and Fig. 2 are then readily made. The *sone* is a unit of loudness whose numbers are determined in the same way as the ordinates in Fig. 1, except that the loudness units begin at 40 phons on the abscissa scale rather than at 0 phons. The loudness level of 40 phons has a certain convenient usefulness. Above the 40-phon level, the loudness-phon curve is nearly straight and represents the more useful portion of the curve of Fig. 1. Again, the 40-phon level of Fig. 1 has been proposed as the reference level for determining the pitch of tones.

The relations between intensity and loudness actually are more involved than Fig. 1 and Fig. 2 indicate. Thus, in many instances, increases in loudness have been found to require intensity changes different from equal decreases in loudness. The numerical differences have not yet been determined satisfactorily, but appear to be fairly large in some instances.

If one desires to have a simple mathematical relation between loudness and intensity, the relation $L = e^{0.076N}$, where L is the loudness and N is $10 \log I/I_0$, fits the facts very well for the middle-range frequencies. This equation states that, for all practical purposes, the loudness is proportional to the cube root of the intensity.[13] Many other simplified equations have been proposed for various frequency and energy ranges. Any general mathematical relation for the determination of loudness will give loudness as a function of intensity, frequency, number and spacing of the overtones, and so forth. Therefore, at the present time, graphical methods appear to be the most acceptable way of arriving at workable relations between loudness and intensity for general purposes.

[10] H. Fletcher, *Speech and hearing*, pp. 226–231.
[11] H. Fletcher and W. A. Munson, J. Acous. Soc. Am. **5**, 83 (1933).
[12] S. S. Stevens, Psych. Rev. **43**, 405 (1936).
[13] L. B. Ham and J. S. Parkinson, J. Acous. Soc. Am. **3**, 511 (1932).

Transcendental Mechanics

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ONE of the most important source books for the history of mechanics is Galileo's *Dialogues concerning Two New Sciences*,[1] published in 1638. The "two new sciences" were the strength of materials and the motion of falling bodies and projectiles. The dialogues, however, do not confine themselves strictly to these subjects, and the book contains practically all that Galileo has to say about mechanics.

Published half a century before the *Principia*, and, in fact, four years before Newton's birth, it is interesting to note that Galileo anticipated Newton in several matters of importance. He stated Newton's first law of motion accurately, and described an experiment similar to that of Newton, showing that pendulums of the same length but with bobs of different weight and materials would oscillate with the same period. Our present interest, however, is in his discussion of the laws of falling bodies.

In those days, it appears from the *Dialogues*,[2] a general impression prevailed that a falling body gained speed in proportion to the distance through which it fell, and Galileo himself held this opinion for a time. It appears also that he abandoned this idea, not because of contrary results of experiment, but because deductive reasoning (not without a flaw) had led him to the impossible conclusion that a body governed by this law would fall through a long distance in the same time as through a short one. He therefore abandoned the space-acceleration idea, and considered the possibility that the gain in speed was proportional to the time of fall. He studied this question experimentally by rolling balls down an inclined plane, and found that the speed gained was proportional to the time, and the space passed over proportional to the square of the time.

It will, I believe, be of interest to develop the theoretical consequences of the space-acceleration hypothesis, and obtain a picture of the strange world that it presents to us.

On this hypothesis the fundamental equation is

$$v = v_0 + as, \tag{1}$$

where v_0 is the initial speed, v the final speed, s the distance fallen and a the acceleration due to gravity, in this case of dimension T^{-1}.

Differentiating with respect to t we obtain $dv/dt = av$, the solution of which is $v = Ae^{at}$. Since $v = v_0$ when $t = 0$, it follows that $A = v_0$, and we obtain the general equation connecting v and t,

$$v = v_0 e^{at}. \tag{2}$$

The general equation connecting s and t can be obtained by eliminating v from Eqs. (1) and (2). It is

$$s = (v_0/a)(e^{at} - 1). \tag{3}$$

Equations (1), (2) and (3) define a system of mechanics which, though self-consistent, is of a strange and transcendental character.

Suppose that a body is thrown directly upward with an initial speed v_0. By Eq. (2), in which a is now negative, we see that the speed v decreases with the time according to an exponential law and, in consequence, will not become zero until t is infinite; but the height reached by the body will approach a finite limit, given by Eq. (3), in which a is negative, say, $-k$, and t is infinite. This limit is $s = v_0/k$. A body projected upward will therefore never return to the earth, but will never reach more than a finite height.

Consider the case of a body released at some distance above the earth, and left free to fall under the action of gravity. Equation (2), in which v_0 is now zero, tells us that the body cannot start, but will remain suspended at rest forever. If, however, in some way an initial speed is given the body, it will fall with a continually increasing speed, with no limit to the distance of fall except that set by the earth or some other obstacle in its path.

It is of interest to inquire further into the reason why a body with no initial speed cannot start to fall. It may be suggested that, according to the equation $v = as$, v must be zero as long as s is zero; but does not the same hold good in the

[1] Tr. by Crew and de Salvio (Macmillan, 1914).
[2] Reference 1, pp. 167–168.

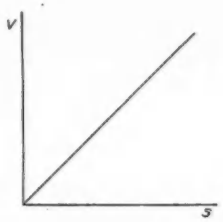

FIG. 1. $v=as$.

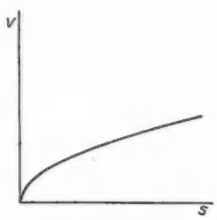

FIG. 2. $v=(2as)^{\frac{1}{2}}$.

corresponding equation of ordinary mechanics, $v=(2as)^{\frac{1}{2}}$?

In order to answer this question we may plot the two curves $v=as$ (Fig. 1), and $v=(2as)^{\frac{1}{2}}$ (Fig. 2). In Fig. 1 the locus is a straight line; in Fig. 2, a parabola. Comparing the values of dv/ds when $s=0$ in the two curves, we see that in Fig. 1 $dv/ds=a$, a finite quantity, while in Fig. 2 it is infinite. Herein lies an important difference between the two cases.

In Fig. 1, at the origin, $dv/ds=a$, a finite quantity. If ds approaches zero, dv must also approach zero, in order that the ratio of the two shall be finite. But at the origin in Fig. 2, the ratio dv/ds is infinite, and if ds is zero, dv may be finite.

Here we have an illustration of the fact that when an infinite value enters into the question, that which is otherwise impossible may be expected to happen; and it is instructive to ponder over the fact that in the system of "practical" mechanics in which we live and move and have our being, the ability of a body to start to fall is in some way dependent upon the abstruse mathematical theorem that zero times infinity may be a finite quantity.

Student Likes and Dislikes in the Elementary Laboratory

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PROBABLY one of the most controversial and widely discussed issues in the teaching of college physics is the manner in which the laboratory should be taught. But if we are to make physics a course that students want to take rather than one they take because it is a part of their program, a study of their likes and dislikes should give us some basis for making changes. The writer does not want to suggest that we should cater to the students to the extent that they miss the rigor and drillwork that is so necessary a part of physics, but rather that their reactions to our present methods be analyzed to see how we may better stimulate their interest.

In the present investigation 150 students in the elementary liberal arts physics laboratories at the State University of Iowa were given questionaries at the end of the first and second semesters of 1939–1940. The first questionary contained a list of the 29 experiments performed during the first semester. The student was asked to give his opinion of each experiment, that is, whether it was good, acceptable or poor. By giving the students three instead of two choices, it was hoped that the students without any very positive opinions would be eliminated. In most cases about 50 percent of the answers given by the students "fell" in the middle class. The second questionary contained the list of experiments performed during the second semester, and also nine questions of such a nature that the student would have an opportunity to express his attitude toward various aspects of the course. To insure more reliable results, the students were asked not to sign their names.

The laboratory manual used by these students was one written specifically for the Iowa laboratories by Professors C. J. Lapp, J. A. Eldridge and D. Colson; the instructions given in this manual tend to be brief. The laboratory part of the course consists of one two-hour laboratory period per week. Each instructor has charge of about 12 students, who generally perform the same experiment at the same time.

The results of the first questionary and of the first part of the second are given in Table I, in

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TABLE I. *Order of preference for experiments.*

(LIKES/DISLIKES) ×10	EXPERIMENT	(LIKES/DISLIKES) ×10	EXPERIMENT
190	Archimedes' principle (5)	21	Electric motor and dynamo (71)
190	Viscosity (17)	20	Reflection and refraction (80)
180	Vernier and micrometer calipers (1)	19	Weather maps (A)
98	Kundt's tube (31)	18	Heat of vaporization (44)
89	Freely falling body (8)	17	Force board (2)
85	Surface tension (18)	17	Resistance of lamps (60, 60*)
77	Bending beams (15*)	16	Ballistic pendulum (11)
73	Specific heat of metals (40)	16	Electromagnetic induction (65)
68	Relative heat efficiency (46)	15	Radio (E)
62	Plotting of magnetic fields (50, 50*)	15	Boyle's law (5)
61	Heat of fusion (43)	15	Young's modulus (15)
48	Linear expansion (41)	15	Resistance measurements, voltmeter and ammeter
46	Housewiring (D)	14	Photometry (84)
43	Wheatstone bridge (62)	12	Fall of potential along a wire (59)
42	Autowiring (C)	10	Concave mirrors (81)
41	Car on track (9)	10	Microscopic determination of refractive index (86)
38	Transformer (70*)	−11	Atwood machine (8*)
32	Focal length of lens (82)	−12	Electrostatics (55)
28	Speed of rifle bullet (11*)	−14	Simple crane (3)
25	Mechanical equivalent of heat (47)	−16	Magnetometer (52)
23	Resonance of air column (32)	−19	Electromotive force of cells (64)
22	Simple machines (13)	−22	Water jet (10)
22	Constants of a galvanometer (61)		

which the 45 experiments are listed in order of preference. In the left-hand column is given the ratio, multiplied by 10, of the number who liked an experiment to the number who disliked it; for example, 190 students liked the experiment on Archimedes' principle to every 10 who disliked it. A negative sign indicates just the reverse ratio. The number in parentheses is the number of the experiment in the laboratory manual used.

It was hoped, at the beginning of the investigation, that the best liked or most disliked experiments could be found to possess certain common characteristics, such as quality of results, understandability, complexity of apparatus, practicality or demonstration of principles; but an attempt to analyze the experiments in terms of such characteristics was only moderately successful. Although it is difficult to isolate any single group of characteristics common to the best liked experiments, the rather general conclusion can be drawn that students liked experiments the results of which can be checked and that yield good results. It is seen that all of the top experiments possess at least one particularly outstanding characteristic; and the presence of this one characteristic, regardless of the presence or absence of any other, seems to be enough to produce in the student a strong liking for an experiment. The most disliked were those that were not outstanding in any respect. For example, the Atwood machine, the magnetometer, electromotive force of cells and the water jet all yield poor results, are not very practical, are hard to understand and are quite complex. Students are less keen about practical experiments than many of us would like to think. The particularly practical experiments are those designated by letters in parentheses, namely, housewiring, autowiring, simple machines, weather maps and the radio. Evidently they are pretty much scattered through the list.

In order to see whether the experiments in any one branch of physics were rated above those in any other, Table II was compiled. It shows that the best liked experiments were, in order of preference, those in mechanics, electricity, light and heat. The probable reason is that in mechanics and electricity there is a wider variation in the type of experiment performed, whereas in heat a certain similarity exists among all of the experiments. The experiments in light are, for the most part, rather poor and apparently uninteresting.

Table III contains a summary of the answers to the questions asked on the second questionary. The number appearing after each answer is the number of students who gave that answer; for example, 35 students thought that practical applications were the most important thing they got out of the laboratory. It is interesting to note that, while the general tendency of teachers is to increase the number of experiments given, many

TABLE II. *Experiments grouped according to branches.*

ORDER OF LIKE	MECH.	HEAT	ELEC.	SOUND	LIGHT
1	5				
2	17				
3	1				
4				31	
5	8				
6	18				
7	15*				
8		40			
9		46			
10			50		
11		43			
12		41			
13			*D*		
14			62		
15			*C*		
16	9				
17			20*		
18					82
19	11*				
20		47			
21				32	
22	13				
23			61		
24			71		
25					80
26		*A*			
27		44			
28	2				
29			60		
30	11				
31			65		
32			*E*		
33	7				
34	15				
35			58		
36					84
37			59		
38					81
39					86
40	8*				
41			55		
42	3				
43			52		
44			64		
45	10				

of the students preferred fewer experiments which may be performed more thoroughly, and an even larger percentage believed this procedure to be more valuable. The reports required of the students at Iowa consist of a statement of the object of the experiment, tabulation and discussion of results and answers to certain questions. The average report runs from 2 to 3 pages. Even so, most students spent from 2 to 2.5 hr writing it up. Almost 10 percent spent more than 3 hr. Perhaps the most disturbing answers are those to the fifth question, which indicate that 27 percent of the students believed that the laboratory was not worth the two hours which they spent on it each week.

TABLE III. *Summary of answers to questions.*

1. *Do you prefer to perform many experiments sketchily or a few thoroughly?*
   Many experiments, 47 Few experiments, 102
2. *Which kind do you think to be most valuable?*
   Many experiments, 12 Few experiments, 43
3. *About how much time on the average did it take you to write up each experiment?*

Hours	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	6.0
No. of students	2	18	18	58	23	18	10	2	1

4. *Do you like the instructor to discuss the experiment in detail before you start to work or would you prefer to begin immediately and discuss difficulties as you come to them?*
   Detailed discussion, 101 Do not prefer detailed discussion, 43
5. *Do you believe that the laboratory has been worth two hours of time each week?*
   Yes, 74 No, 28
6. *What is the most important thing you have got out of the laboratory this year?*
   - Supplementary to lecture, 37
   - Practical applications of physics, 35
   - No answer, 16
   - How to wire electric circuits, 14
   - Experimental technics, 13
   - Learning to write reports, 4
   - Learning to use scientific method, 4
   - Nothing, 4
   - Calculation, 2
7. *What has been your pet peeve?*
   - None, 25
   - Write-ups, 21
   - Poor equipment, 15
   - No excitement, 8
   - Rather work alone, 6
   - No advance assignment, 6
   - Misspent time, 6
   - Writing answers to questions, 5
   - Too much preliminary explanation, 5
   - Tedious calculations, 4
   - Experiments not correlated with classwork, 4
   - Impractical experiments, 3
   - Instructors, 3
   - Experiments done in high school, 2
   - Lack of accuracy, 2
   - Discrepancy between manual and apparatus, 2
8. *What did you like best about the laboratory?*
   - Practical experiments, 23
   - Being able to do something for one's self, 17
   - No answer, 16
   - Informal relationship between instructor and student, 16
   - Makes theory clearer, 10
   - Setting up apparatus, 10
   - Lasts only two hours, 6
   - Equipment, 6
   - Discussion, 5
   - Instructor, 5
   - Putting theory into practice, 5
9. *In what ways do you believe that the instruction could be improved?*
   - Laboratory is satisfactory, 37
   - No answer or suggestions, 30
   - Give detailed and specific instructions, 21
   - Assign experiments a week ahead, 15
   - Correlate better with lectures, 11
   - Start working immediately, 6
   - Smaller laboratory classes, 4
   - Improve laboratory manual, 4
   - More competent assistants, 3
   - Better apparatus, 3
   - Let students work alone, 3
   - More demonstration experiments, 2
   - More practical experiments, 2
   - Shorter and more concise explanations, 2

What then do all these answers imply and what use can we make of them? First, those experiments that were most disliked apparently either should be justified on the grounds of what is good for the student or should be improved. Second, if practical experiments are given, they should be capable of yielding good results that can be readily verified. Third, we should make the instructions for experiments as clear and concise as possible and still not pamper the students too much.

Results such as these are most useful for improving the situation in which they originated and perhaps are not of very general applicability. They are presented here mainly to illustrate the type of study that should form an integral part of every laboratory improvement program; for it is partly by knowing what the student likes and what he wants that we can hope to make physics a more useful and interesting subject in our universities and colleges.

A Simple Discussion of Kepler's Laws

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THE object is to present in as simple form as possible the fundamental properties of the planetary orbit. The motion of a planet around the sun, or of an electron around the nucleus, is an elegant example of the laws of conservation of energy and of angular momentum. While, of course, advanced mathematical preparation is required to understand the fine details of the motion, the more important features can be outlined with the use of merely algebraic and geometric methods.

In the following discussion, therefore, we presuppose that the reader solves quadratic equations, knows the elements of plane trigonometry and understands the familiar textbook derivation of the law of centripetal force F_c,

$$F_c = ma_c = -mv^2/r = -m\omega^2 r = -4\pi^2 n^2 rm, \quad (1)$$

where m is the mass of the particle moving with linear speed v and angular speed ω in the circular orbit of radius r, and n is the number of revolutions per unit time.

We shall assume that the attracting center is fixed and stationary; in other words, that the mass of the sun is infinite.

CIRCULAR ORBITS

Consider first the special case of the circular orbit. The centripetal force will then be exactly equal to the (as yet unknown) gravitational force, or

$$m\omega^2 r = mkR, \quad (2)$$

where kR is the acceleration due to this gravitational force, k is a constant describing the strength of the field, and R is a function describing the way in which the acceleration (or force) varies with the distance r. For a planet revolving about the sun, k is ΓM, where Γ is the gravitational constant and M is the mass of the sun; for an electron in the field of the nucleus, k is Ze^2, where Z is the atomic number of the nucleus, and e is the electronic charge.

Equation (2) may be rewritten in the form

$$\omega^2 r/R = k = \text{const.}$$

Now, for a particle in a circular orbit, Kepler's third (empirical) law asserts that

$$\omega^2 r^3 = \text{const.} \quad (3)$$

Therefore

$$R = 1/r^2, \quad (4)$$

at least for a circular orbit. If we admit that Eq. (4) is valid, no matter what the shape of the planet's orbit, we are led directly from Kepler's third law to Newton's law of universal gravitation.

Let us now determine the energy of the moving particle. If the force F on a particle is $-mk/r^2$, it can easily be shown by algebraic methods[1] that the work required to move the particle from $r=a$ to $r=b$ is

$$mk(1/a - 1/b) = V_b - V_a,$$

where $V_b - V_a$ is the potential energy acquired by the particle as the result of having changed its position. If we let b approach infinity, we have $V_\infty = 0$, and dropping subscripts, obtain

$$V = -mk/r, \quad (5)$$

where V is the loss of potential energy of the particle in falling, under the action of the inverse square force, from infinity to the position r. Thus, in an attractive field, V will be a negative quantity; in a repulsive field, V will be positive.

The kinetic energy T of the particle is $\frac{1}{2}mv^2$. For the *circular orbit only*, this may be written as

$$T = \tfrac{1}{2}m\omega^2 r^2. \quad (6)$$

Combining this with Eqs. (2) and (5), we obtain

$$T = -\tfrac{1}{2}V \quad (7)$$

as the necessary condition for the orbit in an inverse square field to be circular. The total energy E of the particle evidently is

$$E = T + V = \tfrac{1}{2}V = -T. \quad (8)$$

The fact that E is negative means that the total energy of the particle in its orbit is less than its potential energy when at rest at infinity. In other

[1] Spinney, *A textbook of physics* (Macmillan), chapter on "Electrostatics"; Copeland, Am. J. Phys. (Am. Phys. T.) **7**, 314 (1939).

words, work must be done to remove the particle permanently and completely from the attracting field. In atomic physics, the quantity $-E$ is called the *binding energy*.

Let us next consider Kepler's second law: the rate at which the radius vector sweeps is constant throughout the orbit. Now, it may be shown by an argument exactly similar to that used for deriving Eq. (1) that

$$\text{Rate of sweeping area} = \tfrac{1}{2}r^2\omega. \tag{9}$$

Kepler's second law may therefore be written (for *any* position of the particle in *any* orbit in *any central* field):

$$r^2\omega = \alpha = \text{const.} \tag{10}$$

Multiplying the left-hand member of Eq. (10) by the mass of the particle, we obtain the angular momentum of the particle about the attracting center, or

$$mr^2\omega = h = \text{const.} \tag{10'}$$

Thus Kepler's second law states that h, the angular momentum of the particle about this center, is constant throughout the orbit. This means that the force exerted by the attracting center has no component at right angles to the radius vector.

Now, for any given orbit, E, h and k will be constant, and by manipulating Eqs. (6) to (10') we can find expressions for r and ω in terms of these constants; namely,

$$r = h/(-2mE)^{\frac{1}{2}} = mk/(-2mE) = h^2/m^2k, \tag{11}$$

$$\omega = -2E/h = m^3k^2/h^3 = 4hE^2/m^3k^2. \tag{12}$$

Since it will be more convenient from here on not to deal explicitly with the mass m of the planet or other particle under discussion, we shall introduce the symbols,

$$\Phi = V/m, \quad U = T/m, \quad W = E/m.$$

It will also be convenient to have for reference and comparison the following complete list of relations for a circular orbit:

$$\Phi = -k/r = -2U = 2W, \tag{5a}$$

$$U = \tfrac{1}{2}\alpha\omega = \tfrac{1}{2}k^2/\alpha^2 = -\tfrac{1}{2}\Phi = -W, \tag{6a}$$

$$W = \Phi + U = \tfrac{1}{2}\Phi = -U = -\tfrac{1}{2}k/r = -\tfrac{1}{2}\alpha\omega, \tag{8a}$$

$$\alpha = -2W/\omega = k/r\omega = k/(-2W)^{\frac{1}{2}}, \tag{10a}$$

$$r = \alpha/(-2W)^{\frac{1}{2}} = k/(-2W) = \alpha^2/k, \tag{11a}$$

$$\omega = \alpha/r^2 = -2W/\alpha = k^2/\alpha^3 = (-2Wr)^2/\alpha^3 = \alpha(-2W/k)^3. \tag{12a}$$

In a given *noncircular* orbit, W, α and k will be constant, and these equations for the circular orbit should be compared with those that follow.

Noncircular Orbits

By way of introduction to the problem of the noncircular orbit, let us consider two special cases:

Case I. A particle is moving (instantaneously) perpendicularly to the radius vector from the attracting center, but its speed at that instant is such that the centripetal force required to keep it on a circle is somewhat greater (or somewhat less) than the gravitational force actually experienced at that radius.

Case II. A particle is (instantaneously) in such a position with such a speed that at that instant T is $-\frac{1}{2}V$, but its path through that position is not perpendicular to the radius vector from the attracting center.

Consider the first case. Since we are restricting ourselves to those points on the path where the motion is at right angles to the radius vector, we will have for each such point,

$$\tfrac{1}{2}\omega^2r^2 - (k/r) = W, \quad r^2\omega = \alpha. \tag{13}$$

Here we have two simultaneous equations in two unknowns, r and ω. The solutions are

$$r = [k \pm (k^2+2\alpha^2W)^{\frac{1}{2}}]/(-2W) = \alpha^2/[k \mp (k^2+2\alpha^2W)^{\frac{1}{2}}], \tag{14}$$

$$\omega = 2[k^2+\alpha^2W \mp k(k^2+2\alpha^2W)^{\frac{1}{2}}]/\alpha^3 = 2\alpha W^2/[k^2+\alpha^2W \pm k(k^2+2\alpha^2W)^{\frac{1}{2}}], \tag{15}$$

where the upper signs go together. Of course, the mere fact that two values of r satisfy Eqs. (13) does not tell us how the particle travels from one such position to the other; neither does it tell us how many of each such positions there are, nor where they are located with respect to one another; but it does tell us that the particle somehow travels from one of the positions given by the upper sign in Eq. (14) to one of those given by the lower sign, for these are the only positions where the motion can change from r increasing to r decreasing, or vice versa.

Let us digress for a moment to consider some simple properties of the ellipse, which, for the present discussion, may be defined by the

equation

$$(x/a)^2+(y/b)^2=1,$$

where a is the semimajor axis and b is the semiminor axis, and where we have taken the center of the ellipse (the intersection of the axes) as the origin of coordinates. If some point f, distance $-c$ along the major axis from the center, is taken as the origin[2] (Fig. 1), the equation becomes

$$[(x-c)/a]^2+[y/b]^2=1. \tag{16}$$

Let us now choose f so that $c^2=a^2-b^2$. Then the point f is one of the foci of the ellipse, and the

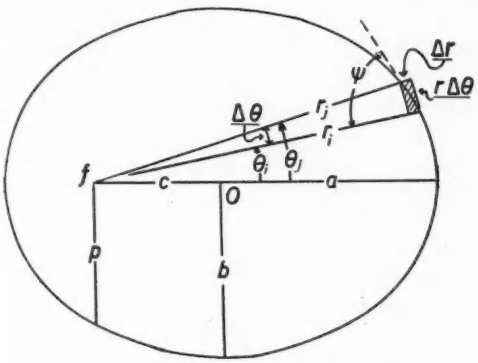

FIG. 1. The ellipse: $v_\theta=r\omega$; $v_r=\lim \Delta r/\Delta t$; $v_s=(v_r^2+v_\theta^2)^{\frac{1}{2}}=v_\theta \csc \psi$.

ratio c/a is the eccentricity ϵ. From this, we see that $b=a(1-\epsilon^2)^{\frac{1}{2}}$. The height p of the ordinate at the focus, called the semi-latus rectum or semiparameter of the ellipse, can be found by making $x=0$ in Eq. (16), giving

$$p=b^2/a=a(1-\epsilon^2).$$

If r is the distance from this focus to either vertex of the ellipse, we have

$$r=a\pm c=a(1\pm\epsilon)=b^2/(a\mp c)=p/(1\mp\epsilon).$$

Comparing this with Eq. (14), we see that the maximum and minimum values of r given there are related to each other in the same way as those of the ellipse for which

$$\begin{aligned} &a=k/(-2W), \quad c=(k^2+2\alpha^2W)^{\frac{1}{2}}/(-2W),\\ &b=\alpha/(-2W)^{\frac{1}{2}}, \quad \epsilon=(k^2+2\alpha^2W)^{\frac{1}{2}}/k.\\ &p=\alpha^2/k, \end{aligned} \tag{17}$$

These expressions for a, b and p should be compared with those given for the radius of the circular orbit in Eqs. (11a).

Of course, this does not prove that the particle actually moves along this ellipse, but it gives us a starting point for further discussion.

We might point out here the following conclusions from Eqs. (17):

(1) All particles in a given field, moving in ellipses having the same major axis, have the same total energy W per unit mass.

(2) All particles in a given field, whose orbits have the same parameter, have the same angular momentum per unit mass. This is true also for the parabolic and hyperbolic orbits mentioned below.

Both of these relations are of importance in the Bohr theory of the atom.

(3) When $\alpha=k/(-2W)^{\frac{1}{2}}$, the orbit is a circle of radius $r=a=b=p$, $(c=0, \epsilon=0)$.

The equations for the motion of the particle at the vertices of this ellipse may be conveniently tabulated here:

$$\begin{aligned} &r=a\pm c=b^2/(a\mp c)=\alpha^2/[-2W(a\mp c)],\\ &\omega=\alpha/(a\pm c)^2=4W^2(a\mp c)^2/\alpha^3,\\ &U=\tfrac{1}{2}\alpha\omega=-W(a\mp c)/(a\pm c),\\ &\Phi=-k/(a\pm c)=2Wa/(a\pm c),\\ &U+\Phi=W=-k/2a=-\tfrac{1}{2}(\alpha/b)^2. \end{aligned} \tag{18}$$

These equations should be compared with the corresponding ones for the circular orbit.

Substitution of $x=r\cos\theta$ and $y=r\sin\theta$ in Eq. (16) yields

$$\begin{aligned} r&=b^2/(a-c\cos\theta)\\ &=\alpha^2/[k-(k^2+2\alpha^2W)^{\frac{1}{2}}\cos\theta]. \end{aligned} \tag{19}$$

Remembering that $r^2\omega=\alpha$ throughout the entire orbit, we get

$$\begin{aligned} \omega&=4W^2(a-c\cos\theta)^2/\alpha^3\\ &=[k-(k^2+2\alpha^2W)^{\frac{1}{2}}\cos\theta]^2/\alpha^3. \end{aligned} \tag{20}$$

So far, we have shown merely that this ellipse is one possible orbit for the particle to take, and not that it is the only possible orbit. Indeed, so far as Kepler's second law is concerned, we might just as well have had

$$\begin{aligned} &\omega=\alpha/(a+c\cos\theta)^2,\\ &r=a+c\cos\theta, \end{aligned}$$

and these relations would satisfy both of Eqs. (13) for $\theta=0$ and for $\theta=\pi$; they would not,

[2] This is the procedure in most books on atomic physics. In celestial mechanics, the other focus is usually taken as the origin, and the polar angle measured from the perihelion, called the *true anomaly*, is usually denoted by v, so that Eqs. (16) and (19) become

$$[(x+c)/a]^2+[y/b]^2=1, \quad r=b^2/(a+c\cos v).$$

however, satisfy the energy equation for any other value of θ. The same may be said of a host of other easily written relations among r, ω and θ; they will satisfy the momentum equation throughout the orbit, but will satisfy the energy equation only at the maximum and minimum values of r. Equations (19) and (20) are the only ones that satisfy the energy equation for all values of θ. We shall now show this for the very simple case, $\theta=\sin^{-1}(b/a)$. As will be seen shortly, this is the position mentioned in *Case II*.

By following an argument similar to that used for deriving Eq. (1) (see the legend of Fig. 1), it will be seen that

$$\begin{aligned} v_s &= \omega r \csc\psi, \\ U &= \tfrac{1}{2}v_s^2 = \tfrac{1}{2}\alpha\omega\csc^2\psi, \end{aligned} \tag{21}$$

where v_s is the linear speed of the particle in its orbit, and ψ is the angle between r and v_s. Consider the particle as it is crossing the minor axis of this ellipse. In this position, $\theta=\sin^{-1}(b/a)$, $r=a$, $\psi=\theta$, $\csc\psi=a/b$ and $\omega a^2=\alpha$. Substituting these expressions into the energy equation, we obtain

$$\begin{aligned} U &= \tfrac{1}{2}\alpha\omega\csc^2\psi = \tfrac{1}{2}(\alpha/b)^2 = -W, \\ \Phi &= -k/a = 2W, \\ U+\Phi &= W. \end{aligned} \tag{22}$$

Thus, $\theta=\sin^{-1}b/a$ is seen to be the position mentioned in *Case II*.

The physical significance of Eqs. (22) is this: if from a point distant a from the attracting center, a number of particles are projected with a velocity $v_s=(k/a)^{\frac{1}{2}}$ in different directions, all of these particles will describe ellipses with the same semimajor axis a, and of eccentricity $\epsilon=\cos\psi$; the angular momentum of each particle will be given by $\alpha=av_s\sin\psi$. The particle projected at right angles to the radius vector will describe the circle of radius a with angular momentum $\alpha=av_s$.

Let us now find the period P of the planet in its orbit. From Eq. (9) and the definition of α, we obtain

$$\alpha P = 2\times\text{area of the ellipse} = 2\pi ab.$$

Elimination of b and then W by means of Eqs. (17) yields $P=2\pi a/(-2W)^{\frac{1}{2}}$, or

$$P^2 = 4\pi^2a^3/k. \quad \text{(Kepler's third law)} \tag{23}$$

This equation should be compared with Eq. (3) for the circular orbit.

It should be mentioned here that, although we have found the velocity of the particle at each position in its orbit, we have not found the time at which it arrives at that position. To find this requires the solution of a transcendental equation, called Kepler's equation. However, there is a simple graphical solution of this problem given in many of the textbooks on astronomy.

So far, we have considered only those orbits for which W is negative. When $W=0$, several of the equations derived become indeterminate. Perhaps the best procedure is to start all over again with Eqs. (13), set W equal to zero, solve these equations and follow the same steps as those taken in the case of the elliptic orbit, remembering that the semi-parameter of *any* orbit in an inverse square field is $p=\alpha^2/k$. We then see that the minimum radius vector is related to p in the same way as in the parabola

$$y^2 = p(2x+p),$$

whose polar equation is

$$r = p/(1-\cos\theta) = \alpha^2/k(1-\cos\theta).$$

When W exceeds zero, two cases may be considered: the particle moves in an attractive field, as in the case of certain comets; or the field is repulsive, as in nuclear scattering experiments, in which case $\Phi=+k/r$. In either case, by following the procedure previously outlined, we find that the path is one branch or the other of the hyperbola for which

$$\begin{aligned} a &= k/2W, & c &= (k^2+2\alpha^2W)^{\frac{1}{2}}/2W, \\ b &= \alpha/(2W)^{\frac{1}{2}}, & \epsilon &= (k^2+2\alpha^2W)^{\frac{1}{2}}/k. \\ p &= \alpha^2/k. \end{aligned}$$

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BROWN University will continue in each semester of the year 1941–42 the special program of advanced courses, seminars, and research in mechanics which it is sponsoring during the current summer session. Persons can profit from the instruction only if they have a considerable knowledge of physics and have the equivalent of at least one year of graduate work, including rigorous courses in mechanics, advanced calculus and differential equations. Because of subventions, there will be no charge for tuition; a few fellowships with stipends up to $600 are also available. For application blanks, address the Dean of the Graduate School, Brown University, Providence, Rhode Island.

The Electric Field Associated with a Steady Current in Long Cylindrical Conductor

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THIS problem, by contrast with its magnetic counterpart, is ignored in nearly all textbooks on electricity and magnetism.[1] As a consequence, the writer finds that even among advanced college students, a statement that an electric field exists outside of a conductor carrying a steady current is usually received with some skepticism. If the student has already given thought to the matter, he argues that, inasmuch as the magnetic field is steady in the neighborhood of such a current, there can be no accompanying induced electric field. Moreover, since the current is steady everywhere along the conductor, no element of the conductor can acquire an excess of electric charge of either sign. Consequently there is no electric intensity associated with any charge distribution which is directly traceable to the current.

J. H. Poynting[2] neglected the normal component of the electric intensity in the neighborhood of a straight conductor carrying a steady current and, as a consequence, found that the energy-stream vector is perpendicular to the wire. Heaviside[3] showed that when the normal component of the electric field is taken into account, the energy-stream vector is very nearly parallel to the wire "with a slight slope toward the wire." However, he did not give the mathematical analysis of the problem.

We shall now present the solution of the problem of the electric field intensity inside and outside of a solid cylindrical conductor along which the current is steady as an interesting example of Laplace's equation. In order to prevent distortion of the field owing to the return portion of the closed circuit, we shall assume that the current returns to the generator along a hollow, perfectly conducting cylinder coaxial with our conductor. It is well known that the hollow space inside of such a return conductor is entirely free from electric and magnetic fields associated with the flow and any static charges that might reside upon the surface. The current in the inner solid conductor is now solely responsible for the field, the hollow outer conductor merely serving to confine the propagated energy within the hollow space. Let the radii of the inner and outer cylinders be r_0 and r_1, respectively. We shall use the cylindrical coordinates z, r, with the origin at the end connecting plate joining the inner and outer cylinders. At any point in the field the equation which must be satisfied by the potential is

$$\frac{\partial^2 V}{\partial r^2}+\frac{1}{r}\frac{\partial V}{\partial r}+\frac{\partial^2 V}{\partial z^2}=0. \tag{1}$$

Let us take a solution of Eq. (1) in the form $V=F(r)Z(z)$, where F is a function of r only and Z is a function of z only. This enables us with the help of Eq. (1) to put

$$\left(\frac{\partial^2 F}{\partial r^2}+\frac{1}{r}\frac{\partial F}{\partial r}\right)\Big/F=-\frac{\partial^2 Z}{\partial z^2}\frac{1}{Z}. \tag{2}$$

As the left-hand member of Eq. (2) is independent of z and the right-hand member is independent of r, both members must be equal to some constant. This constant is zero, for any other value is incompatible with the conditions of the problem. For example, if the constant be real and positive, Z is periodic in z, which is, of course, ruled out by the condition that the potential must be a linear function of z in order that it may satisfy Ohm's law along the conductor. Hence

$$\frac{\partial^2 Z}{\partial z^2}=0, \quad \frac{\partial^2 F}{\partial r^2}+\frac{1}{r}\frac{\partial F}{\partial r}=0,$$

which give the solutions,

$$Z=Az+B, \quad F=\alpha \log r+\beta,$$

[1] A very brief discussion is given by Franklin and Macnutt in *Advanced electricity and magnetism* (1915), pp. 254–258.

[2] J. H. Poynting, "On the connection between electric current and the electric and magnetic inductions in the surrounding field," Trans. Roy. Soc. London, **A176**, 278 (1885).

[3] Heaviside, *Electrical papers* (Macmillan), vol. I, pp. 346–349, vol. II, pp. 94–95.

and finally for any point in the field,

$$V=(Az+B)(\alpha \log r+\beta).$$

We shall distinguish the values of V for space inside and outside by means of the subscripts 1 and 2, respectively. The boundary conditions are that V is continuous at the boundary of the conductor; the tangential components of E_1 (the electric intensity) are equal on both sides, and the difference between the normal components of D on the two sides is equal to $4\pi\sigma$, where σ is the surface density of charge. Since r may take the value 0 on the inside, we must assume $\alpha=0$; β may be taken as 1 without any loss of generality and, because $V=0$ when $z=0$, we have $V_1=A_1z$ on the surface and inside of the conductor. The electric intensity in the conductor is given by

$$-\partial V_1/\partial z=E_1=-A_1=RI,$$

where I is the current and R is the resistance per unit length of the conductor. There is no component of the electric intensity perpendicular to the surface on the inside.

For the space outside the conductor, V is given by

$$V_2=A_2z(\alpha \log r+\beta). \qquad (3)$$

We now apply the boundary conditions to evaluate A_2 and β, α being taken as 1 without any loss of generality:

$$-(\partial V_2/\partial z)_{r=r_0}=-A_2(\log r_0+\beta)=E_1; \qquad (4)$$

at the return conductor,

$$(\partial V_2/\partial z)_{r=r_1}=0,$$

which gives $\beta=-\log r_1$, and, on substituting this value of β in Eq. (4), we obtain

$$A_2=-E_1/\log (r_0/r_1).$$

Thus Eq. (3) becomes

$$V_2=-\frac{E_1z}{\log (r_0/r_1)}\log (r/r_1). \qquad (5)$$

For all points inside the conductor,

$$V_1=-E_1z, \qquad (6)$$

which shows that the field inside is uniform and parallel to the conductor. Equation (5) gives the components of the field intensity in the hollow space:

$$(E_z)_2=E_1 \log (r/r_1)/\log (r_0/r_1)$$

and

$$(E_r)_2=E_1z/r \log (r_0/r_1).$$

These expressions show that for a long line, that is, z large, the ratio $(E_r/E_z)_2$ is very large near the surface of the conductor. This confirms the statement that the electric intensity near the surface is very nearly perpendicular to it.

We are now able to determine the way in which the surface charge is distributed along the conductor:

$$(\partial V_1/\partial r)_{r=r_0}-(\partial V_2/\partial r)_{r=r_0}$$
$$=4\pi\sigma=E_1z(1/r_0)/\log (r_0/r_1)$$

or

$$\sigma=E_1z/4\pi r_0 \log (r_0/r_1).$$

We can also find the capacitance per unit length of the two coaxial cylinders. The charge per unit length is $2\pi r_0\sigma$, and the potential difference between the opposite elements of the capacitor at any place is E_1z. Hence the capacitance per unit length is $1/2 \log (r_0/r_1)$, which is the familiar formula usually given for the case of two coaxial cylinders having uniform charge distributions of opposite sign.

The foregoing expressions for the electric field intensity show that the Poynting energy density vector is radial within the conductor but has a longitudinal as well as a radial component on the outside. The longitudinal component varies inversely as the square of the distance from the wire, for each of the two factors—electric and magnetic intensities—whose product determines its value varies inversely as the distance from the conductor. The radial component of the Poynting vector determines the rate at which the wire receives energy from the field; this energy is dissipated as heat. The longitudinal component measures the energy current directed through the dielectric toward whatever receiving apparatus the line contains, the function of the wire being that of a guide for the energy stream.

You cannot teach a man anything, you can only help him find it within himself.—GALILEO

An Apparatus for Investigating the Variable Specific Heat of Carbon

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ALTHOUGH it is well known that the specific heat of a substance varies with the temperature, an apparatus designed to measure the variation is seldom found in undergraduate laboratories. Yet the existence of such variations might well receive some emphasis in the intermediate laboratory, particularly since a simple apparatus capable of measurements to 5 percent can easily be constructed from materials costing less than two dollars.

APPARATUS

A central section of the hollow carbon cylinder is shown in Fig. 1. The carbon, selected because

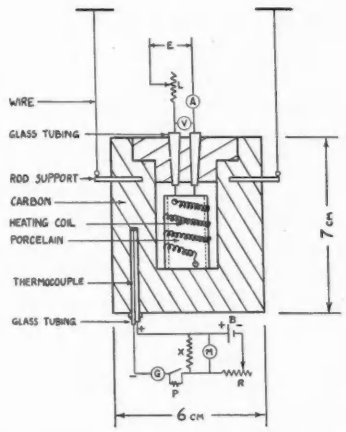

FIG. 1. Carbon cylinder and circuits: $E=110$ v a.c. or d.c.; $L=200$ ohms; A, 0–10 amp; V, 0–100 v; $B=1.5$ v; $X=1.0$ to 0.1 ohm; M, 0–30 mv; G, galvanometer; P, protective resistance; $R=400$ ohms.

its specific heat varies markedly from room temperature to 500°C, should weigh at least 350 gm to insure sufficient thermal capacity. Inside the cylinder is mounted a Nichrome wire heating coil of about 15 ohms resistance. It is desirable to minimize the mass of foreign materials in the cylinder.

One thermocouple placed in a hole in the center of the cylinder wall is sufficient since the temperature gradient is uniform throughout, provided the cylinder is of uniform wall thickness. It is convenient to use a Chromel-Alumel thermocouple because its emf-temperature relationship is practically linear from 0°C to room temperature, and therefore its cold junction correction is easily computed.

OPERATION

It is advisable to work in a fairly large room with no drafts. First, maximum safe power, about 500 w, is supplied to the heating coil. After the thermocouple emf has reached a steady value (equilibrium temperature), the power is shut off and the cylinder is allowed to cool to room temperature; meanwhile readings of thermocouple emf and time are taken. A typical cooling curve is shown in Fig. 2, where the thermocouple emf has been converted to temperature.

Next, any fixed amount of power is supplied to the coil, and the cylinder is allowed to reach its equilibrium temperature. Readings of equilibrium thermocouple emf and power input are

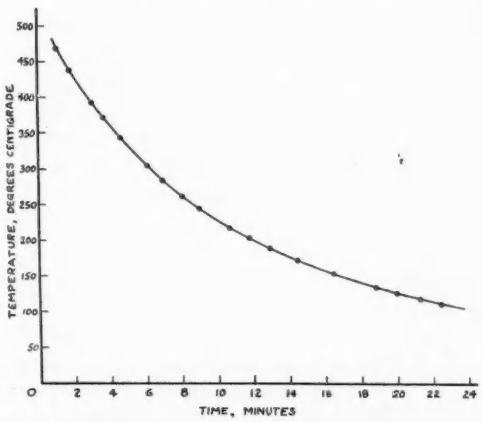

FIG. 2. Cooling curve for a 338-gm carbon cylinder. Room temperature, 25.8°C. Correction has been made for the cold junction of the thermocouple.

* Now at Palmer Physical Laboratory, Princeton University, Princeton, New Jersey.

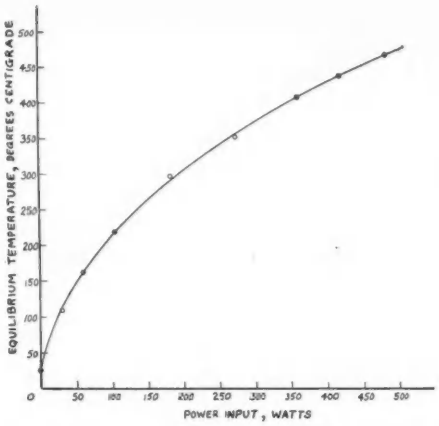

FIG. 3. Equilibrium temperature curve. The carbon cylinder was at room temperature for zero power input.

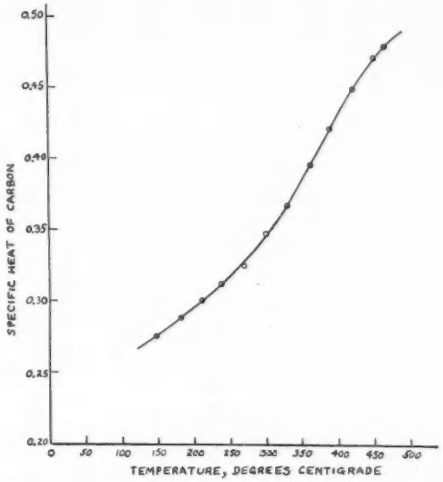

FIG. 4. Specific heat of carbon as a function of temperature. The correction for foreign material has been made.

taken for this and other values of power input. A curve such as that in Fig. 3 is then plotted, where the thermocouple emf has again been converted to temperature. It is desirable to have both the cooling and equilibrium temperature curves embrace the same temperature range.

CALCULATION OF RESULTS

The defining equation for the thermal capacity of a substance, $\Delta Q = ms\Delta\theta$, may easily be put into a form that is applicable to this apparatus, namely,

$$dQ/dt = ms\,d\theta/dt, \qquad (1)$$

where dQ/dt is the rate of loss of heat, m and s are the mass and specific heat of the carbon, and $d\theta/dt$ is the rate of change of temperature. At any particular equilibrium temperature dQ/dt equals the power input to the coil. The slope of the tangent to the cooling curve at this same temperature equals $d\theta/dt$ and, since m is known, s may be computed. Values of s may be obtained for all temperatures common to the cooling and equilibrium temperature curves, and a curve showing the variation of the specific heat of carbon with temperature may be drawn. Figure 4 is typical.

In addition to the cold junction correction for the thermocouple, a correction for foreign material in the cylinder is made, by subtracting $s_x m_x/m$ from the values of s obtained in Eq. (1), where m_x is the mass of the heater and s_x is its weighted mean specific heat. This correction, which may be as large as 5 percent, is not exact since s_x is assumed constant over the temperature range of the cylinder. If desired, a radiation correction may be applied.

I am indebted to Dr. Robert D. Huntoon who offered invaluable suggestions regarding the design and construction of this apparatus.

Scientific inquiries in recent years suggest that excellence of the college product is not so much a function of college equipment or diversified curriculum as of other things—things much more simple and less expensive: careful selection of staff and student body, a budget that is balanced from the standpoint not only of the controller but of the student—WALTER A. JESSUP

The Integrating Sphere

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THE gap between the work of the illumination engineer and the elementary laboratory exercises on the determination of candlepower is, unfortunately, a wide one and needs to be bridged in two ways, one theoretical and the other practical. The first difficulty lies primarily in the fact that, although our concept of brightness is an elementary one from the psychological point of view, the physical units in which the quantities can be measured are derived; the common definitions are based on the former and not the latter considerations. From the practical point of view the chief difficulty has been the lack of apparatus of the integrating-sphere type for student use, although such devices of engineering accuracy have been available for years. An inexpensive form of this apparatus has been in use in our laboratory for several years and has proved sufficiently satisfactory to warrant a description. Perhaps a brief review of the theoretical complications is worth while as an introduction to the advantages and disadvantages of the apparatus.

From the physical point of view the fundamental quantity in the study of illumination is *radiant energy*. The eye is affected not by the total amount of such energy but by the rate at which it is received, or the *power*, usually measured in watts. Moreover, the eye is affected unequally by equal amounts of power in different wave-length regions; it offers no response to wave-lengths less than 4×10^{-5} cm or greater than 7×10^{-5} cm, approximately; the response within this region for the normal eye is given by the luminosity curve which has its maximum at about 5.55×10^{-5} cm, in the yellow green. Hence the defining equation for *total luminous flux* is written as

$$F=c_{\max}\int_0^\infty K_\lambda P_\lambda d\lambda,$$

in which K_λ is the luminosity factor, ranging in value from 0 to 1 and given by the luminosity curve, P_λ is the radiant flux for the wave-length interval $d\lambda$, and $c_{\max}$ is the constant of proportionality, whose value for the normal human eye has been carefully determined as 621 when F is in lumens and P is in watts. If this equation had been taken originally as the defining equation, doubtless the units would have been chosen so that this constant would have been 1. However, the lumen was defined in terms of the original intensity standard, the *candle*. A candle made in a certain way, burned at a certain rate and viewed in the horizontal direction provides the standard of light intensity; it is essentially a measure of luminous flux per unit solid angle, not a simple unit, although the concept of brightness seems simple itself. The *lumen* may then be defined as the luminous flux such that the intensity viewed from a certain direction is the same as that of a standard candle viewed horizontally; or the definition may be stated in terms of a hypothetical source: If a source is uniform in all directions and in any direction has the same intensity as a standard candle, its luminous flux is 4π lumens.

The practical need for the shift from standards of luminous intensity to the measurement of total luminous flux is obvious when one considers the difference between the use of light sources in direct methods of lighting and in indirect methods; for, if all of the light emitted is to be made use of by means of reflectors and diffusing globes, the total luminous flux is the essential quantity, and not the luminous intensity in a single direction.

The one additional piece of apparatus necessary for the introduction of the student to the use of these concepts is the integrating sphere. The idea involved is by no means new; it was suggested by Sumpner in 1892, and its use for photometry was developed by Ulbricht a few years later. It can be proved that, if the inner surface of a sphere is coated with a perfectly diffusing surface of high reflectivity and no selectivity in the visible range, and if there is introduced into this a light source of infinitely small dimensions, the infinite number of reflections suffered by every ray of light will result in an illumination that is equal over the entire

inner surface of the sphere. Under these conditions the brightness of this inner surface depends on the radius of the sphere, the reflectivity of the surface coating and the total luminous flux from the light source. Hence this flux can be studied by cutting a small hole in the sphere and studying the luminous intensity of this window, provided only that the window is shielded from the direct rays of the lamp. The laboratory problem then resolves itself into the construction of a sphere as nearly like this ideal sphere as possible, one large in comparison with the size of the lamp and the window, and with an inner surface as nearly as possible perfectly diffusing and white. Such spheres designed for the use of illumination engineers have been available for years, but at a price that is prohibitive for undergraduate use. The small one here described is a compromise with accuracy, but this seems justifiable if its use means a correct introduction to important concepts.

Two hemispheres of spun aluminum, 16 in. in diameter and $\frac{1}{16}$ in. thick, were obtained;[1] one hemisphere was made with a $\frac{1}{2}$-in. flange so that as the second was slipped into this a snug fit was assured. Small holes 180° apart were drilled in each so that the spheres could be bolted together (Fig. 1). In one hemisphere a window 2 in. square was cut and so placed that when the sphere was on its stand the window would be at right angles to the photometer bench. In the other hemisphere a 4×4-in. door was cut, and was attached by a hinge and fastened with a hook. In what was to be the bottom of this hemisphere was drilled a hole $\frac{7}{16}$ in. in diameter. A 12-in. length of $\frac{1}{8}$-in. pipe was threaded at one end to fit a heavy laboratory stand and also for about 2 in. in the center so that curved flanges could be screwed against the hemisphere both inside and outside and thus hold it firmly; the upper end was threaded and on this a lamp socket, threaded to fit, was screwed. The socket had a standard holder which was used to support a screen about 5 in. high and 4 in. wide, curved at the bottom to fit the holder and placed to screen the window from direct light from the lamp. In assembling, the wires to the socket were threaded through the pipe, and the hemisphere was attached to the pipe by means of the flanges. The entire inside of the hemispheres, including the screen and outside of the socket, were painted with flat white and, after this was dry, with a thick paste of magnesium oxide and water. Although the underlying coat insures a light surface in case the oxide flakes off, it yellows with age and so is not sufficient alone. The hemispheres were assembled and any dark or flaked spots were retouched through the door. It is necessary to obtain a lamp rated in lumens. We have found these only in large sizes, which is unfortunate since the sphere itself is comparatively small. The window may be covered with a diffusing surface such as tracing paper; ours has been used both that way and with no cover at all. The latter would be impractical if the photometer room were dusty or if on inspection the light from the window did not appear well diffused.

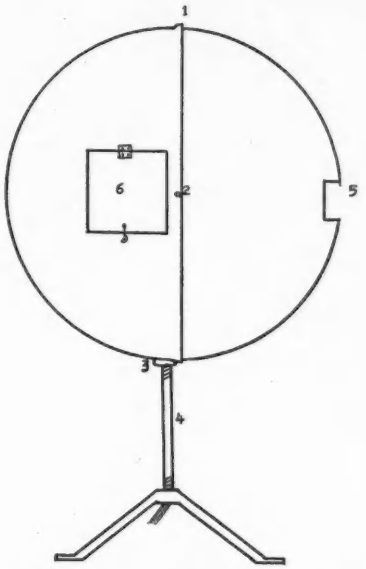

FIG. 1. The integrating sphere: 1, flange; 2, bolt; 3, curved flanges for holding the hemispheres together and to the support 4; 5, window; 6, door.

The size of the sphere was chosen so that it could be used at one end of the photometer bench already available, a two-track bench fitted with a Lummer-Brodhun head. This head and the comparison lamp were adjusted to the same vertical height above the bench as the window of

[1] From the Teiner-Torngren Co., 44 Garden St., Boston. The cost, including the basic machining, was $16.25 in 1937.

the sphere. For the comparison lamp, any type of incandescent lamp can be used; it is connected so that its voltage can be controlled, as with a potentiometer. The standard lamp in the sphere was connected to a wattmeter. The brightness of the fields of the photometer head are balanced in the usual way. When this has been done, the luminous intensity of the window is known in terms of an arbitrary standard. The standard lamp is then removed from the sphere and an unknown inserted in its place, and the photometric balance again obtained. The ratio of the luminous flux from the lamps then equals the ratio of the luminous intensities of the window in the two cases, and the total luminous flux emitted by the unknown lamp can easily be calculated.

If the experiment is to be extended to the study of sources of extensive area,[2] a comparison lamp of known horizontal candlepower should be employed, so that the absolute luminous intensity of the window is known. Then the new source, such as a fluorescent bulb, is mounted in place of this comparison lamp and is shielded so that an area comparable to that of the sphere window is visible. The photometric balance is found as before, and so the luminous intensity in candles is measured and the candles per sq cm calculated. If the surface is of approximately the same brightness in all directions, the total luminous flux emitted by the lamp can be found by using the formula πBA.

Students find it interesting to compare the efficiencies in lumens per watt of ordinary frosted tungsten bulbs, daylight bulbs and fluorescent lamps. The color differences are an obvious source of error, but results are reproducible within a few percent. A source of larger error lies in the use of a large lamp in a sphere of a size suitable for miniature lamps. At present little can be done beyond pointing out this error, but it is to be hoped that standardized small lamps will soon be made available. The substitution of a square opening as a light source and the use of the inverse square law and its hypothetical point source introduce still another error; if the greatest dimension of the window is $\frac{1}{5}$ the distance from window to photometer head, this error is about 1 percent. As this is smaller than either of the other errors mentioned it is scarcely worth while to reduce it further. It may be interesting to point out, as a measure of relative accuracy, that the engineering standard for such apparatus is to keep the dimensions of the extended source less than 1/20 of its distance to the photometer head.

Eye Sensitivity

One experiment with the Lummer-Brodhun photometer has been developed for the sake of those students who are more interested in the physiology and psychology of vision than in the engineering type of illumination problems. It is not a short experiment but offers some possibility for investigation. The object is to test the accuracy of the statement that the response of the eye to variations in intensity of monochromatic light differs from its response in case the light is heterochromatic. A sodium lab-arc provided the monochromatic light. The heterochromatic source was the light from a tungsten lamp filtered in such a way as to give as good a color match as possible with the sodium light. We used the light from the window of the integrating sphere since this was easy to shield, but any source of constant luminous intensity would do. A set of Wratten Experimental Filters[3] were used in obtaining the best visual match, filter No. 16 being chosen. Care must be taken to exclude scattered light. The luminous intensity of the monochromatic light was found in terms of that of the heterochromatic light, then both intensities were reduced in equal steps by use of Eastman neutral density screens of densities 0.1, 0.2, 0.5 and 1.0. The two screens of density 0.1 were used, one with each light source, and the balance obtained; then the

[2] In order to study extended sources, such as fluorescent lamps, one must make use of the relation between the luminous intensity per unit area and the luminous flux. It can be proved that, if an illuminating surface is perfectly diffusing, that is, if it appears equally bright from all directions, the Lambert cosine law applies to this surface; and in such cases the total luminous flux F is given by the expression πBA, in which B is the luminous intensity per unit area in candles per square centimeter (πB is then expressed in lumens per square centimeter, or lamberts) and A is the area of the total emitting surface in square centimeters. See Hardy and Perrin, *The principles of optics*, Chapters 1 and 13; also the chapter on photometry in the forthcoming edition of Millikan, Roller and Watson, *Electricity, light and atomic physics*, which the author has seen in manuscript.

[3] Eastman Kodak Co., Rochester, New York.

screens were interchanged and the balance obtained again. It is desirable to have enough filters so that several cross checks of these densities can be made, unless the densities can be measured independently. We found that, although the screens supposedly alike were quite accurately so, the addition of 0.1, 0.2, 0.2 did not equal 0.5. When the eye was dark-adapted throughout the experiment, the determinations of the luminous intensity of the monochromatic light in terms of the heterochromatic light were found to remain constant, even for very low light intensities. But when the eye was light-adapted, reductions in the intensity resulted in increased sensitivity to the heterochromatic light, owing to the shift from cone to rod vision with a corresponding shift of sensitivity toward the shorter wave-lengths transmitted by the filter. This was accompanied by a change in color that was noticeable but not sufficient to make a photometric match very difficult or to make one of the other filters in this set a better match. As the amounts of light transmitted by the filter at different wave-lengths are given in the *Wratten filter book* and the relative amounts emitted at these wave-lengths by a tungsten lamp run at a certain voltage are also obtainable, a good deal of information is available to the interested student for interpretation.

If much student work is to be done with a set of filters of gelatin such as these, it is wise to mount them. The glass and the Scotch tape used for mounting Kodachrome slides are both inexpensive and convenient for this purpose.

A Laboratory Experiment on Diffusion of Gases

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THERE are a number of excellent qualitative demonstration experiments on the diffusion of gases.[1] However, few, if any, lend themselves to quantitative measurements. The phenomenon is an important link in molecular theory as well as an excellent example of the many cases wherein the rate of change of a quantity is proportional to the quantity itself.

The apparatus shown in Fig. 1 is simple and permits quantitative measurements. It consists of a 1-l (1150-ml) flat-bottom flask with a wire hook cemented to the bottom to permit it to be hung in an inverted position from one arm of a beam balance. A wire loop is also attached to the neck so that it may be hung upright. The flask was first counterbalanced when filled with air. After being filled with hydrogen, it was again counterbalanced by placing small weights on the flat portion of the flask. These weights and the time were recorded, and the balance was gently lowered onto its supports. Subsequent weighings were made at convenient known intervals.

That the added weights are proportional to the hydrogen in the flask may be seen as follows.

Let

M = total mass of air to fill flask;
M_0 = total mass of gas to fill flask;
x = fraction of mass of gas in flask at time t;
m = mass added to balance system at time t;
m_0 = mass added to balance system when flask is filled with gas.

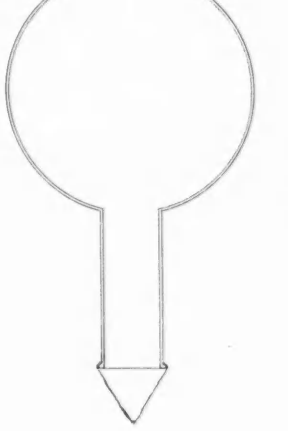

FIG. 1. Arrangement of flask for measurement of the diffusion of hydrogen.

[1] Sutton, *Demonstration experiments* (1938), p. 466.

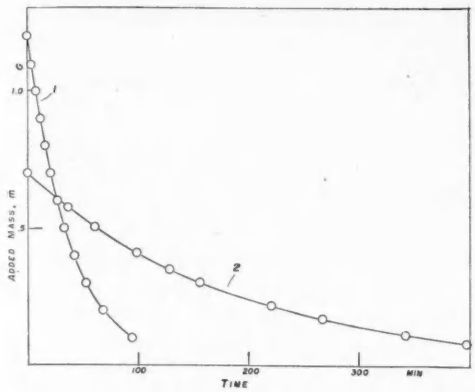

FIG. 2. Buoyant force of gas in flask as a function of time: *1*, hydrogen in inverted flask; *2*, carbon dioxide in upright flask.

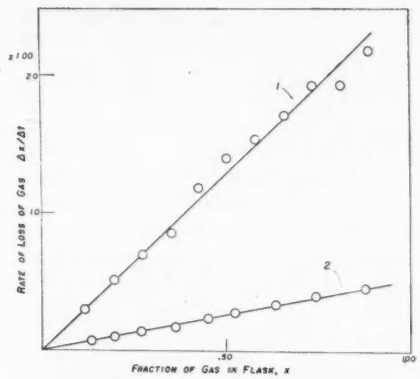

FIG. 3. Rate of loss of gas from flask as a function of fraction in flask: *1*, hydrogen; *2*, carbon dioxide.

Then, at any moment,

$$(1-x)M+xM_g+m=M,$$

so that

$$m=x(M_g-M).$$

Also, since $m_0=M_g-M$, $x=m/m_0$.

Further, if the rate of loss of gas from the flask is proportional to gas in the flask, then $\Delta x/\Delta t=Dx$, where D is the diffusion constant for the arrangement. Obviously this equation may be integrated and the verification consists of showing that $\log x$ is proportional to t. Because such a treatment is not possible in a similar case if the observable, x, includes an unknown zero correction—the asymptote—and because the differential form seems to the students to be closer to what takes place, the author prefers an analysis following that form. It gives the diffusion constant quite as accurately as the other form and also gives the asymptote, if any exists.

Table I contains a portion of the observed data for hydrogen and their analysis. The value of m_0 was found to be 1.290 gm by calculation from the measured volume of the flask and by a subsequent direct measurement with hydrogen. Curve *1*, Fig. 2, is a graph of m as a function of t. It is to be noted that about 80 percent of the hydrogen diffuses out in less than 1 hr, so that the experiment may be used in the classroom as well as in the laboratory. Curve *1*, Fig. 3 is a graph of $\Delta x/\Delta t$ as a function of the mean value of x. It is seen that these data fall reasonably well along a straight line, the slope of which is about 0.0250 min^{-1}. It might be noted that a graph of $\log x$ as a function of t gives a similar straight line with the observed points rather crowded at the upper end (small values of t).

TABLE I. *Data for hydrogen.*

TIME t (MIN)	MASS m (GM)	FRACTION x	$\Delta x/\Delta t$	MEAN x
0	1.20	0.931		
			−0.0223	0.892
3.5	1.10	.853		
			− .0192	.814
7.5	1.00	.776		
			− .0195	.737
11.5	.900	.698		
			− .0171	.660
16.0	.800	.621		

Similar data were obtained for carbon dioxide after the flask had been hung from the balance in an upright position. The values of m were smaller than those for hydrogen owing to the smaller ratio of the molecular weight to that of air. Curve *2*, Fig. 2, is a graph of the observed data; it is seen that the rate of diffusion is much smaller than that of hydrogen. Curve *2*, Fig. 3, is a graph of $\Delta x/\Delta t$ as a function of x for carbon dioxide. Again the points fall along a straight line which goes through the origin. The diffusion constant for carbon dioxide is about 0.0053 min^{-1}.

Graham's law requires that the diffusion constants should be inversely proportional to the square roots of the molecular weights of the two gases. The ratio of the diffusion constants is 4.73, while the square root of the ratio of the molecular weights of carbon dioxide and hydrogen is 4.70.

Construction of Thermodynamic Models for Elementary Teaching

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THE author has found great value in the use of solid models in freshman physics teaching to illustrate (1) the law of ideal gases, and (2) the van der Waals equation of state and the behaviors of vapors and liquids. Many freshman students have difficulty in visualizing the shape of the thermodynamic surfaces from, let us say, a blackboard sketch of a set of isotherms, and the better part of a precious lecture hour may be consumed in explaining, with many words, what would be almost obvious if one had a solid model in his hands. The author has found that more than half of a class of average intelligence, having learned that an ideal gas is described by the equation $PV=NRT$, will be sorely puzzled by the adiabatic equation for ideal gases, $PV^{\gamma}=\text{const}$. It helps these puzzled students surprisingly little to be told that, since the first equation contains three variables, it is not inconsistent with the second. With many of them, however, the matter becomes clear when they are shown an ideal gas surface on which have been plotted isotherms, adiabatics, and so forth.

The models shown in Figs. 1 to 4 were constructed by the author. It is probable that a model like the ideal gas surface of Fig. 1 can be purchased from supply houses,[1] but it is not at all difficult to construct. The model of the van der Waals surface, however, contains features which the author believes to be novel.

The ideal gas surface shown in Fig. 1 is made of plaster of Paris. Three square boards form the main supporting frame. These are joined with nails and glue in the shape of the three square surfaces which meet at one corner of a cube. As can be seen, the outward corner of one surface is cut away, leaving a profile in the form of an equilateral hyperbolic arc. The dihedral angles adjacent to this arc are terminated with pieces of wood in the form of right triangles as shown. To reduce the weight of the model, it is better not to fill the box, so formed, full of plaster but rather to construct a backing of tar paper and wire netting nailed onto a few triangular wooden braces to form a surface roughly approximating the plaster surface but so situated as to give room on top of it for a coating of plaster $\frac{1}{2}$ to $\frac{3}{4}$ in. thick. Before nailing down the tar paper and wire netting, the woodwork should be painted inside and out with several coats of paint to protect it against absorbing moisture from the plaster and thus to prevent warping. On the visible outer surfaces of the wood, it is desirable to use "blackboard" paint which, when dry, gives a surface suitable for chalk marks indicating numbers, symbols, coordinates, and so forth. Two round holes not visible in Fig. 1 should be bored in the board farthest from the observer. These serve to ventilate the space behind the plaster, thus facilitating the drying, and also may be used as grip holes into which the fingers may be inserted for conveniently carrying the model.

The plaster may be mixed a little at a time and applied in several coats, without too much delay for hardening between applications. When sufficient plaster has been built up, the ideal gas surface is very easily generated by the use of a straight bar of rectangular, cold rolled brass or

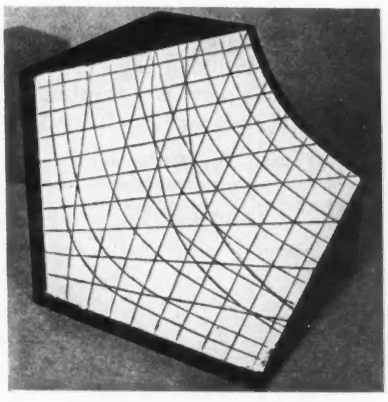

FIG. 1. The ideal gas surface.

[1] See also: Verwiebe, "Models of thermodynamic surfaces," Am. J. Phys. (Am. Phys. T.) **3**, 179 (1935); Zemansky and Herman, "The Gibbs and Mollier thermodynamic surfaces," Am. J. Phys. (Am. Phys. T.) **4**, 194 (1936).

steel. This bar is held parallel, let us say, to the surfaces of constant pressure and caused to move so that near one end it slides up and down the vertical (pressure) axis, while near the other end it follows first the hyperbolic arc and then the hypotenuse of the right triangle. The bar in so doing successively assumes the positions of all the lines of constant pressure shown in the photograph. As the bar moves, it is made to cut away the excess plaster until the surface has the correct shape. It is, of course, possible also to move the bar so that it assumes the positions of successive lines of constant volume, and it is preferable to test the figure of the surface both ways before leaving it to the hardening process. A basin of water should be kept handy to wash off the excess plaster from the bar. After the plaster has hardened, the isotherms are scribed with a machinist's surface gauge. For this operation, the model should preferably lie on that one of its three flat faces from which the hyperbolic arc was cut, and this face should lie on a large plane surface upon which the surface gauge can slide. The lines of constant pressure and of constant volume, since they are straight, can readily be scribed with a straight edge, but a surface gauge may also be used conveniently for them. The grooves so scribed can then be filled in with pencil, paint or India ink applied to the plaster. Two or more adiabatic curves, as shown in Fig. 1, should also be plotted using the adiabatic equation. The coordinates defined by the straight lines of constant pressure and of constant volume are used as reference coordinates for this purpose. The adiabatics here shown were plotted for a value $\gamma = 5/3$.

The van der Waals surface was not made of plaster because the removable blocks which form an important feature (about to be described) have, unfortunately, some rather sharp knifelike edges, which would be too fragile if plaster were used. This surface was therefore built up out of boards glued together with the grain running vertically in alternate boards and horizontally in the remainder (Fig. 2). The flat surfaces of the boards are parallel to planes of constant temperature. The boards are first cut out in the form of true square slabs and the various isotherms are traced, one on each board. The profiles of the isotherms are then sawed out with a band saw, and it is advisable to save *both* pieces. It is convenient for plotting to use the van der Waals equation in the "reduced" form,

$$(P+3/V^2)(V-\tfrac{1}{3})=8T/3,$$

in which the variables P, V and T are pure numbers measuring, respectively, pressure, volume and temperature referred to critical pressure, critical volume and critical temperature as unity. If the boards are about $\frac{1}{2}$ in. thick, a good result will be obtained by plotting an isotherm for every 1/20 of critical temperature from $T=0$ to $T=24/20$. In Fig. 1, the isotherms represented with white (painted) lines on the black surface (finished with "blackboard" paint) are spaced at intervals of 1/10 of critical temperature. When the boards have been sawed along the isothermal profiles, the pieces which are to form the model are coated with glue and nailed together in correct register. Any excess glue remaining in the steps between isotherms should be carefully removed. The remaining pieces of the boards are then stacked up without glue so as to nest loosely into the steps on the model, the whole forming a roughly cubical block. This is then clamped tightly between plane wood or metal sheets of sufficient stiffness to exert a uniform pressure all over and insure that the model will dry with the isothermal surfaces plane. It is a good idea to insert sheets of paper between the pieces of board which are later to be removed after the

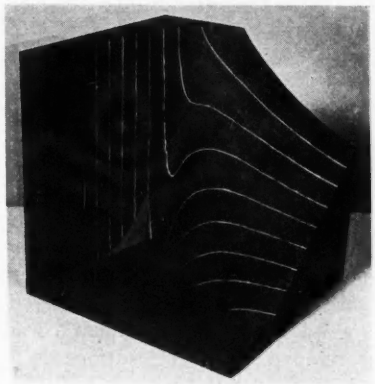

FIG. 2. The van der Waals surface.

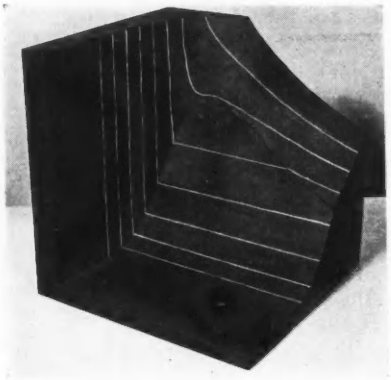

FIG. 3. Model with the second block in place.

glue has dried, to prevent any excess glue which has squeezed out under pressure from sticking to them. The stepped approximation to the van der Waals surface must now be dressed down with a wood file until the salient corners are entirely removed and the surface is just in contact with the plotted isotherms. Fortunately, the van der Waals surface is a "ruled surface"; all its profiles along lines of constant volume are straight lines. The wood file, therefore, should be held so that it strokes the model along these straight lines of constant volume. It is well to select a wood file with a "half round" cross section and to employ the round side. A "rat-tail" file of rather small diameter will be required for finishing the valley corresponding to the sharply pointed minima of the isotherms. In the model of Fig. 2, the plane representing zero volume is the vertical surface of the model at the back to the left, and the thickness of this vertical slab at the front edge (in the plane of zero temperature) represents the "covolume" which is one third of the critical volume. The slabs on which the isotherms were plotted were so dimensioned that the correct thickness to give this "covolume" could be obtained by gluing a flat sheet of wood on the face (in Fig. 2, invisible on the left-hand back side) so as to cover the end grains of the slabs and strengthen the model. As can readily be seen, the laminated structure of the model need only extend down to $T=0.3$, or 14 board thickness below critical temperature, the remainder of the left-hand wall being filled out with a flat piece of wood dressed down with the file to a continuation of the van der Waals surface. The model is eventually glued to a base board, shown in Fig. 2, whose upper face is the plane of zero pressure. Before this is done, however, a band-saw cut must be made to remove that portion of the model which fails to express the behavior of a substance when two phases coexist. This cut is faintly visible in Fig. 2. Unless the nails have been very judiciously placed in the laminated structure, it will be safer to make this cut with a metal band saw.

To determine the profile of this cut (projected on the V, T plane), it is convenient to have the entire set of isotherms plotted on a sheet of paper to the same scale as that adopted for the model. Indeed, this plot should be made before plotting the isotherms on the wood, and they can then be traced on the wood slabs directly from the plot by means of carbon paper. It is well known that the region of each isotherm over which two phases (vapor and liquid) can coexist is a straight horizontal line of such height (pressure) that the loop of the van der Waals isotherm *below* the line includes the same area as the loop *above* the line. The pressure corresponding to the height of this line is the boiling pressure for that isotherm. These lines can very quickly be located in height on the paper plot by trial-and-error testing of the equality of the areas with a planimeter. The volumes at the two termini of each line are then read off for each temperature (isotherm) and this immediately gives the necessary data for plotting the profile of the cut projected on the V, T plane. The cut surface both on the removable block and on the

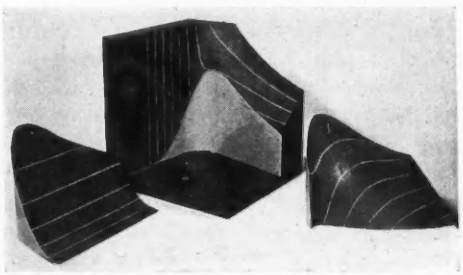

FIG. 4. Model with the two blocks removed.

remainder of the model is painted red to distinguish it from the van der Waals surface. A second block is now prepared to fit against the cut surface of the model and this one is provided with an outward surface such that the isotherms in the two-phase region are horizontal. This is shown in place in Fig. 3. When the model is shown to a class, it can be pointed out that the profile of this second block projected in the P, T plane gives the curve of vapor pressure as a function of temperature, or the boiling curve. This affords a good opportunity also to illustrate the behavior of the pressure as the substance is heated at constant volume, the discontinuity in slope at the cut being very evident. The meanings of the critical point and of the critical isotherms become very clear when the model is viewed with this block in place, as does also the idea of the continuity of liquid and vapor states.

Figure 4 shows the model with the two blocks removed. The first-mentioned block representing the van der Waals equation has a portion on the left-hand side so thin that it is scarcely possible to represent it even in wood. As can be seen in Fig. 2, the absence of this portion leaves a small quasi-triangular area of the cut uncovered, a minor defect which is readily forgiven by the students.

Reproductions of Prints, Drawings and Paintings of Interest in the History of Physics

17. Frescos in the *Tribuna di Galileo*, Part I

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California Institute of Technology, Pasadena, California

THE three frescos in the compartments of the domed ceiling of the innermost room of the *Tribuna di Galileo*[1] depict three momentous periods in GALILEO's life—the rising, the zenith and the setting of his genius. They are the work of LUIGI SABATELLI.

In the first, reproduced in Plate 1, the young student is watching intently a swinging lamp in the Cathedral of Pisa (the inclined flame shows that the lamp is actually swinging). The face of the young GALILEO is very expressive and the position of the hands—a device previously used by RAPHAEL—indicates that he is following a train of thought. So absorbed is he that his hat has fallen from his hand and is seen lying near his feet. According to the well-known story (probably apocryphal) GALILEO timed the swings with the only watch he possessed, his own pulse, and thus discovered the isochronism of

PLATE 1. Galileo discovering the isochronism of the pendulum. [From the fresco in the *Tribuna di Galileo*.]

[1] See Reproduction 16 of this series.

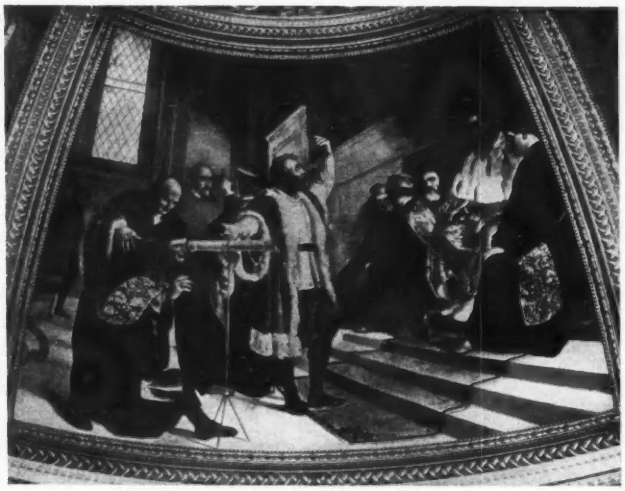

PLATE 2. Galileo presenting his telescope to the Republic of Venice. [From the fresco in the *Tribuna di Galileo.*]

PLATE 3. Galileo imparting his discoveries in dynamics to Torricelli and Viviani. [From the fresco in the *Tribuna di Galileo.*]

the pendulum, a discovery that led later to his design for a pendulum clock.

In the second fresco (Plate 2) GALILEO is shown presenting his newly invented telescope to the Doge and Senate of Venice and explaining how the Republic could make use of the invention (this he actually did on August 24, 1609). One of the Senators is not able to restrain his impatience to look through the telescope and three others are grouped behind him, one waiting to succeed him and the other two speculating about the possible uses of the instrument.

The third painting (Plate 3) shows GALILEO, old and blind, communicating his last ideas on dynamics to TORRICELLI and VIVIANI, the last two and the most loved of his disciples. GALILEO is seated, his left hand placed on a terrestrial globe, and his right indicating the importance of the ideas he is expounding, while TORRICELLI, seated near him, and VIVIANI, youthfully on his feet, listen to him with intelligent attention.

The word "impossible" is to a scientist much like a spur to a horse.—WILLIAM D. COOLIDGE

The Effectiveness of Problem Solving in Producing Achievement in College Physics

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MODERN technics in measuring achievement have shown that many of the generally accepted ideas having to do with learning and classroom performance are poorly substantiated. On many occasions the Committee on Tests of the American Association of Physics Teachers has been unable to discover why certain questions that cover theory which is supposedly well presented in most textbooks were consistently missed by students over the nation, while other questions that pertain to theory which is supposed to be difficult and poorly taught were found to be easy when measured by actual student performance. New technics of classroom instruction, such as the sound motion picture, have come, as has also the necessity of handling larger classes in many institutions. The demand for more transfer of learning by the subject fields served by physics and the new searching for basic laws of learning and transfer have fully justified the demand that the time-honored methods be subjected to evaluation.

The present study deals with the evaluation of problem solving as an achievement-producing technic in college physics as measured by the physics achievement tests prepared by the Committee on Tests of the Association.

The writer's college physics class for the college year of 1937–1938 consisted of 184 students divided into approximately 15 percent freshmen, 45 percent sophomores, 25 percent juniors, 12 percent seniors, and 3 percent graduate students. About 55 percent were premedical students, while the others were a mixture from predentistry, pharmacy, law, commerce, physical education and liberal arts. All had high school physics and about 15 percent had some mathematics in college. This class met for lecture in two sections at eight and nine o'clock Monday, Wednesday and Friday, and for one two-hour laboratory period each week. The entire textbook, *College Physics*, by J. A. Eldridge, was studied without omission. Six problems were assigned as a part of each lesson provided this many were available in the text. The class was told at its first meeting that problems would be assigned as part of each lesson but were not to be handed in —that problem solving was considered to be a mind-developing and achievement-producing activity.

The conduct of the 1938–1939 class differed essentially only in that the solved problems were handed in each day at the beginning of the hour, and were graded and returned through a post office box system at the following class period. At the beginning of the semester the students were informed that their problem work would constitute one fifth of the semester grade. The textbook and problem assignments, the lecturer, the lecture demonstrations, the laboratory exercises, and the method of conducting the classes were as nearly identical for the two years as could be made without being meticulous about minor details, such as the exact order in which lecture demonstrations were given in any single lecture.

Group Matching

Before entering the University of Iowa, each freshman takes entrance examinations in high school content, reading comprehension, reading rate, mathematics aptitude, English training and foreign language aptitude. A composite of these

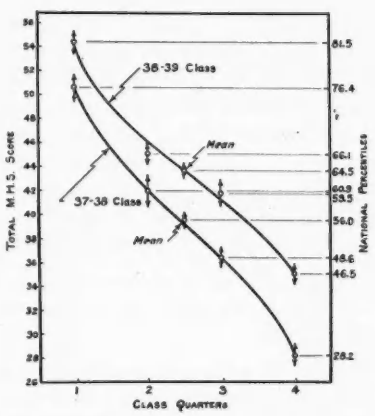

Fig. 1. Class achievement by quarters.

TABLE I. *Group matching statistics.*

GROUP	NO. OF INDIVIDUALS	PHYSICS APTITUDE *Q* SCORE AV.	INTELLIGENCE *Q* SCORE AV.	COMPOSITE *Q* SCORE AV.	STANDARD DEV., COMPOSITE
1937–38	134	10.43	13.16	23.59	9.6
1938–39	134	10.51	13.08	23.59	10.0

TABLE II. *Total group achievement.*

GROUP	MECHANICS, HEAT, SOUND	NATIONAL PERCENTILE	STANDARD DEV.	PROBABLE ERROR S.D.	PROBABLE ERROR OF DIFF.	CRITICAL RATIO
1937–38	39.63	56.0	12.45	0.72	1.04	3.9
1938–39	43.72	64.5	12.79	0.74		

TABLE III.

GROUP	MECHANICS, HEAT, SOUND AV.	NATIONAL PERCENTILE	STANDARD DEV.	PROBABLE ERROR S.D.	PROBABLE ERROR OF DIFF.	CRITICAL RATIO
			Highest quarter			
1937–38	50.70	76.4	10.5	1.23	1.70	2.18
1938–39	54.39	81.5	10.0	1.17		
			Second quarter			
1937–38	42.12	60.9	11.6	1.34	1.68	1.8
1938–39	45.12	66.1	8.83	1.02		
			Third quarter			
1937–38	36.53	48.6	8.04	0.92	1.62	3.35
1938–39	41.8	59.5	11.58	1.34		
			Lowest quarter			
1937–38	28.24	28.2	9.33	1.11	1.53	4.5
1938–39	35.09	46.5	9.08	1.06		

scores gives each student an intelligence rating in percentiles. In the present study these percentiles are turned into *Q* scores.[1] Thus if a student belonged to the least intelligent 5 percent his *Q* score is 1 and if he belonged to the most intelligent 5 percent his *Q* score is 20.

Each student in this study also took the *Iowa Physics Aptitude Test*, revised form A. These scores were also used in the *Q* form and will be known as the *aptitude Q scores*. All of the aforementioned tests have been shown to be highly reliable and valid measuring instruments.[2] The correlation coefficient between the aptitude scores and semester grades in college physics is about 0.60, while the correlation coefficient between the intelligence score and semester grades is about 0.55.

The sum of the intelligence and aptitude *Q* scores (called the *composite score*) was taken as a basis for matching, and 134 pairs of students were found which made up the two groups. Of these, 121 pairs had identical composite scores, while the other 13 pairs were matched by taking two or three at a time from each group. Table I shows how carefully the two groups were matched. The standard deviations also show that the students had a wide range in ability.

The achievement of the two groups was measured by the tests on mechanics, heat and sound prepared by the Committee on Tests of the American Association of Physics Teachers.[3] These particular tests were chosen because they are known to be well balanced, free from errors, highly reliable and valid.

The results of the group achievement appear in Table II and the corresponding comparisons for

[1] A *Q score* is one taking 5 percentiles at a time. The full range is from 1 to 20 instead of 1 to 100 as in percentiles.

[2] G. D. Stoddard, "Iowa placement examinations," "*Iowa studies in education*," vol. III, p. 103.

[3] Later published as *Cooperative Physics Tests for College Students*, Form 1938 A.

the four quarters are shown in Table III. Figure 1 shows the results in graphical form.

These results were obtained using random statistics. Had the difference between the scores of the two groups been small, a more exact evaluation would have been made using matched-pair statistics,[4] which, on data such as these, give probable errors about two thirds as large as those given above.

Summary

(1) The achievement of two matched groups of college physics students from classes in two separate years has been compared. The materials and technics of instruction were practically identical, except that the six problems per lesson were handed in by one class and counted 20 percent of the semester grade while the working of problems by the other class was a matter of conscience only.

(2) The matching of student pairs was done by Q scores weighting equally a battery of freshman intelligence tests and physics aptitude scores.

(3) Physics achievement was measured by the *Cooperative Physics Tests*, mechanics, heat and sound, 1938 A.

(4) Random statistics were used in evaluating the data.

(5) The class that solved and handed in problems was distinctly superior as a group and also at each quarter ability level, the superiority ranging from 18 percentile points in the lowest quarter to 5 percentile points in the highest quarter. The average achievement of the problem-solving group was 10.3 percent greater, which represented 8.5 percentiles on the national scale.

The writer expresses his appreciation to Mr. Stanley Atchison and Mr. John West, research assistants, for many tedious hours of labor required in gathering and evaluating these data.

[4] E. F. Lindquist, *Statistical analyses for educational research* (Houghton Mifflin), pp. 58–59.

Reprints of Survey Articles and Committee Reports for Class Use

Reprints of Survey Articles for Class Use. *American Journal of Physics* (Columbia Univ., New York). These reprints are available only in lots of six or more copies, the price indicated being that for six copies, postpaid; stamps will be accepted in payment: P. Morrison, *Introduction to the theory of nuclear reactions*, 60 cts.; M. S. Plesset, *On the classical model of nuclear fission*, 35 cts.; R. T. Birge, *The propagation of errors*, 30 cts.; J. C. Hubbard, *Ultrasonics*, 50 cts.; W. H. Michener, *A brief table of meter-kilogram-second units*, 15 cts.; R. P. Johnson, *Solid fluorescent materials*, 40 cts.; D. E. Wooldridge, *The separation of isotopes*, 40 cts.; C. W. Ufford, *Spectroscopy*, 45 cts.; M. Randall, *Electrolytic cells*, 60 cts.; Stearns and Froman, *Cosmic rays*, 60 cts.

Reprints of Committee Reports. *American Journal of Physics* (Columbia Univ., New York). The price indicated is for a single copy, postpaid; stamps will be accepted in payment: A.A.P.T. Committee, *Proposal to standardize letter symbols*, 10 cts.; A.A.P.T. Committee, *Suggested four-year curriculum leading to a major in physics*, 5 cts.; A.P.S. Committee, *Physics in relation to medicine* (1923), 10 cts.; *Teaching of physics for pre-medical students* (1937), 5 cts.

NOTES AND DISCUSSION

A Medium-Voltage Regulated D.C. Power Supply

AUSTIN R. FREY
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HUNT and Hickman have published a comprehensive analysis of various types of thermionic voltage regulator circuits.[1] Since their article does not give design data, a brief description of a unit used in our laboratories as a substitute for B batteries may be of interest. The circuit (Fig. 1) is a modification of the Hunt-Hickman degenerative stabilizer. The dimensions of the complete unit, including shielding, are 6×10×14 in., and the total cost of parts, including tubes, is approximately $20.

The unit is a.c. operated, no biasing or other batteries being required. The output potential is continuously variable in two ranges: the lower, from zero to about 160 v; and the upper, from about 160 to 350 v. The maximum output current is approximately 50 ma on the lower range, and 80 ma on the upper range. The hum level is low enough to permit use of the unit in bridge measurements of vacuum tube characteristics.

The stabilization ratio varies with the output potential and load, being about 10 to 1 for most conditions; that is, a variation in line voltage from 110 to 120 v results in a variation in output voltage of approximately 1 v. The effective output resistance is also a function of the output, but is in general less than 25 ohms, and for some loads is zero or slightly negative. The variation in output potential with load is therefore less than that generally observed with B batteries.

Since the characteristics of the type 874 voltage regulator tubes vary to some extent, some slight modifications in the resistances indicated may be necessary in order to secure a satisfactory overlap between the two output ranges. This applies particularly to resistors R_5, R_6 and R_8. Increasing R_5 produces a general decrease in voltage over the low range but has no effect on the high range. Increasing R_6 increases the voltage in both ranges and restricts the extent to which the voltage can be controlled by R_7. Increasing R_8 reduces the voltage in both ranges and also restricts the control of R_7.

The author wishes to thank Dr. P. L. Bayley, of this department, for his assistance and helpful criticism during the design, construction and test of this regulator.

[1] Hunt and Hickman, Rev. Sci. Inst. **10**, 6 (1939).

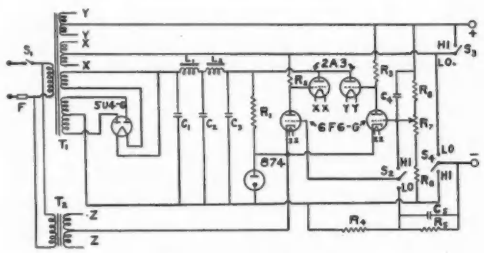

FIG. 1. Circuit and list of parts.

T_1—*Primary*, 115 v, 60 cycle sec^{-1}; *secondary*, 2.5 v, C.T. 3 amp; 2.5 v, C.T. 3 amp; 800 v, C.T. 150 ma; 5 v, 3 amp
T_2—*Primary*, 115 v, 60 cycle sec^{-1}; *secondary*, 6.3 v, C.T. 2 amp
L_1, L_2—8 h, 150 ma, 100 ohms
C_1, C_2—2 μf, 600 v, working
C_3—4 μf, 600 v, working
C_4, C_5—1 μf, 400 v, working

R_1—40,000 ohms, 10 w
R_2, R_3—500,000 ohms, 1 w
R_4—1500 ohms, 10 w
R_5—1350 ohms, 10 w
R_6—85,000 ohms, 1 w
R_7—50,000 ohms, 1 w (potentiometer controlling output voltage, wire wound)
R_8—45,000 ohms, 1 w

F—2-amp fuse

S_1—120 v, 5 amp
S_2, S_3, S_4—Ganged together to form triple-pole double-throw range changing switch.

A Demonstration of Fog Production

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THE usual method of producing fog for demonstrations involves the raising and lowering of a vessel of water to secure the required changes in pressure. All too often the demonstrator must apologize for the very tenuous results. For several years the writer has been using a very simple device which produces dense fogs.

A small amount of water is placed in a 1-l flask and air under a pressure of from 10 to 30 lb/in.^{2} is admitted to the flask through a rubber hose. The hose is connected to a glass or metal tube fitted to a single-hole rubber stopper in the flask. The compressed air quickly blows the stopper out of the flask, whereupon a dense white fog forms when smoke nuclei are present. If air is again forced into the flask the fog will disappear. By holding one hand on the stopper until after the air-intake valve has been closed, the operator can control both the timing of the explosion and the amount of compression. If continuous pressure is exerted on the stopper by hand, it becomes a vibrating valve and the foregoing operation is repeated at short intervals.

When the flask is placed in the beam of a projection lantern it appears to glow and darken on each expansion and compression. The difference in fog production with and without smoke nuclei may be easily observed by a large audience.

Quantitative Treatment of the Racing-Roller Demonstration

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University of California, College of Agriculture, Davis, California

A COMMON means of demonstrating the effect of moment of inertia upon rotation is to employ two disks, identical in size and mass and differing only in their moments of inertia.[1] One side of each disk is painted so that the composite construction may be hidden from the students prior to discussion of the effects demonstrated. When the disks are released simultaneously and one is observed to reach the bottom of an incline before the other, some members of the class will believe that the effect is due to a difference in mass. While this may be disproved by actual weighing, the effect can be made still more convincing by using another pair of disks, much smaller in diameter and mass, but of the same composite construction.

Let the large pair of disks be represented by M and N and the small pair by m and n, where M and m have heavy cores and N and n light cores. From these one may choose two disks in such a way that the acceleration of the large one will be greater than, less than, or equal to that of the small one. It is then possible by a quantitative discussion to show that the acceleration of a rolling object on a given incline depends only upon the ratio of its radius of gyration to the radius upon which it rolls.

Suitably large effects are obtained when the disks are made of lead and wood enclosed in a steel ring. By choosing the radius of the core to be $\frac{1}{2}\sqrt{2}$ times the outer radius R of the disk, the volume of wood and lead is made the same in each disk of a pair. This furnishes a simple means of equalizing the masses of the two disks. Let k be the radius of gyration of a disk and let σ equal k/R. As a disk rolls down the incline, it loses potential energy and acquires kinetic energy of translation and rotation. The linear speed it acquires after having rolled a distance s along an incline of inclination θ is given by

$$v^2=2gh/(1+\sigma^2)=2gs\sin\theta/(1+\sigma^2), \tag{1}$$

and the linear acceleration is therefore

$$a=g\sin\theta/(1+\sigma^2). \tag{2}$$

The value of σ for the disks described may be computed from their moments of inertia. Let ρ_1 be the density of the core and ρ_2 the density of the outer material. Then, *provided the core radius is* $\frac{1}{2}\sqrt{2}R$,

$$\sigma=\tfrac{1}{2}\left(\frac{(\rho_1/\rho_2)+3}{(\rho_1/\rho_2)+1}\right)^{\frac{1}{2}}. \tag{3}$$

If σ_N is the value for disk N and σ_M the value for disk M, then $\sigma_N>\sigma_M$ and, from Eq. (2), $a_M>a_N$. By turning the disks so that their painted sides are away from the class, thus revealing their construction, the foregoing deduction is readily verified. Furthermore, since the construction of the small disks is the same as that of the large ones, $\sigma_n=\sigma_N$ and $\sigma_m=\sigma_M$. Hence $a_m=a_M$ and $a_n=a_N$, and the other combinations follow.

The time required for a disk to roll down a distance s along the incline is given by

$$t=[2s(1+\sigma^2)/g\sin\theta]^{\frac{1}{2}}. \tag{4}$$

If the time of travel for two disks is the same, then

$$s_1/s_2=[1+\sigma_2^2)/(1+\sigma_1^2]. \tag{5}$$

This shows that, for a given length of plane, the distance of separation of the two disks is independent of the angle of inclination. For demonstration purposes, therefore, a long plane with small angle of inclination is desirable.

The foregoing equations, with the exception of Eq. (3), apply to all rolling bodies, provided R in the definition of σ is the radius upon which the body rolls. For a thin ring $\sigma=1$ and such a ring would have a smaller linear acceleration than the composite disks. For homogeneous cylinders and spheres the values of σ are $\frac{1}{2}\sqrt{2}$ and $\sqrt{\frac{2}{5}}$, respectively. For disks rolling on small axles, σ may become much larger than unity.

If allowed to roll up a second incline, all these bodies rise to the same height, irrespective of their masses and accelerations; this provides a useful demonstration of conservation of energy.

Since this experiment involves the concepts of kinetic energy of translation and rotation, and the principle of conservation of energy, it could be used to advantage as a laboratory exercise. Students may derive Eqs. (1), (2) and (4), and check them by measuring the time required to traverse equal distances along the incline.

[1] Sutton, *Demonstration experiments in physics* (McGraw-Hill, 1938). Exp. **M-161.**

A Note on the Nature of Light

Theodore Jorgensen, Jr.
The University of Nebraska, Lincoln, Nebraska

IN the study of its propagation, light is usually considered to have properties of a wave disturbance of some kind, and, in the study of its interaction with matter, light is usually considered to be of a corpuscular nature. It is of interest to make the assumption that light has at the same time properties of both a wave disturbance and a stream of corpuscles. If this is done, it follows logically that the energy of a light corpuscle is proportional to the frequency of the light wave.

Consider a beam of monochromatic light of frequency ν and energy density A falling on a blackbody of unit area. In unit time energy Ac is absorbed by the blackbody; here c is the speed of light. If the corpuscular density of the beam is N, then Nc corpuscles are received by the blackbody in unit time. The energy per corpuscle delivered to the blackbody is thus A/N. Assume that this quantity $E[=A/N]$ is the same for all corpuscles of the beam and is some function $f(\nu)$ of the frequency of the light. This function is to be determined.

Consider next that the blackbody is moved toward the light source with a speed u. An increased amount of energy appears in the blackbody per second. Part of this increase is the result of the extra energy swept up per second from the beam and part of it arises in the work done per second by the force moving the blackbody forward against the light

pressure. Since the pressure of a light beam on an absorbing body expressed in absolute units is numerically equal to the energy density of the beam, this latter source of energy delivers the amount Au per second to the blackbody. The amount received by the blackbody from the beam is $A(c+u)$ per second. The total is thus $A(c+2u)$. The number of corpuscles received per second by the blackbody is $N(c+u)$. Now to an observer at rest relative to the moving blackbody all energy received by the blackbody appears to come from the light beam. With this interpretation the energy per corpuscle received by the blackbody has thus increased, and the frequency of the light beam as seen by the observer moving with the blackbody has likewise increased owing to the Doppler effect. The new frequency ν' is equal to $\nu[1+(u/c)]$. Assume that the energy per corpuscle is the same function of the frequency of the light beam as before. Thus

$$A(c+2u)/N(c+u)=f(\nu').$$

If the speed of the blackbody is small compared with the speed of light, so that $(u/c)^2$ can be neglected in comparison with (u/c), this equation may be written:

$$\frac{A}{N}\left(1+\frac{u}{c}\right)=E\left(1+\frac{u}{c}\right)=f(\nu')=f\left(\nu\left[1+\frac{u}{c}\right]\right),$$

or

$$\left(1+\frac{u}{c}\right)f(\nu)=f\left(\nu\left[1+\frac{u}{c}\right]\right).$$

This functional equation has a solution, $f(\nu)=b\nu$, where b is a constant which can be determined only by experiment. The restriction that the blackbody move slowly compared with the speed of light need not be made if relativity theory is used.

Thus it has been shown that, on the assumption that light has at the same time properties of both waves and corpuscles, the energy associated with a corpuscle is proportional to the frequency of the wave. This is the same result that Einstein found by studying the entropy of cavity radiation and assuming that the radiation acts like a gas.

Activities of Association Chapters

District of Columbia and Environs

The annual meeting was held at George Washington University on April 19, 1941. About 35 persons were in attendance. The members were luncheon guests of the University at the Faculty Club.

Sixteen papers were presented at the morning and afternoon sessions as follows:

A Simple Resonance Demonstration. G. D. Rock, *Catholic University.*

Projection of Interference Fringes. T. B. Brown, *George Washington University.*

Devices for Taking Fear out of Physics. Vola P. Barton, *Goucher College.*

A High School Assembly Program for Relating Physics to Athletics. G. M. Koehl, *McKinley High School* and *George Washington University.*

Aviation Problems in Physics. E. W. Thomson, *U. S. Naval Academy.*

Fluid Drive. C. A. Beck, *Catholic University.*

Experimental Studies of Kinetic Theory. H. Argo, *George Washington University.*

Interpretation of Photoelasticity. J. A. Wise, *U. S. Naval Academy.*

The Nature of Piezoelectricity. K. F. Herzfeld, *Catholic University.*

Demonstration of Transients with Cathode-Ray Oscillograph. T. B. Brown, *George Washington University.*

Fundamental Interference Experiments with Elementary Apparatus. F. E. Fox, *Catholic University.*

Two-Tube Direct Current Amplifier. D. Brumbaugh, *George Washington University.*

Study Facilities of the Library of Congress. L. M. McKenzie, *U. S. Patent Office* and *George Washington University.*

Photographic Diffraction Gratings. A. May, *Catholic University.*

Officers for next year were elected as follows: W. L. Cheney, George Washington University, *President;* W. A. Kilgore, Wilson High School, *Secretary-Treasurer;* E. W. Thomson, U. S. Naval Academy, G. A. Koehl, McKinley High School, and G. D. Rock, Catholic University, *Executive Committee.*

Western Pennsylvania and Environs

A joint meeting with the Pennsylvania Conference of College Physics Teachers was held at St. Vincent College on April 18–19, 1941. More than 60 members and guests, representing about 30 institutions, were in attendance.

The following invited and contributed papers were heard:

Axes, Ellipsoids, Hyperboloids. T. D. Cope, *University of Pennsylvania.*

Photometry and Our Physics Textbooks. A. G. Worthing, *University of Pittsburgh.*

Relation of Physics to Philosophy. O. A. Grosselin, *St. Vincent College.*

Cavitational Luminescence. C. H. Allen, *Pennsylvania State College.*

High School Size as a Factor in Accomplishment in College Physics. H. W. Knerr, *Pennsylvania State College.*

Musical Scales and the Chromatic Stroboscope, with Demonstrations. C. Williamson, *Carnegie Institute of Technology;* W. Fink, *William F. Hammond Music Store.*

Fluorescent Materials for Television and Lighting. B. S. Ellefson, *Hygrade Sylvania Corporation.*

A Triode Model for Use in the Electrolytic Tank. C. Williamson and E. M. Pugh, *Carnegie Institute of Technology.*

The Teaching of the Use of Units. G. Q. Lefler, *Kent State University.*

Early History of Westmoreland County. F. Fellner, *St. Vincent College.*

Georgian Chant—An Illustrated Lecture. C. Zech, *St. Vincent College.*

Other features of the meeting were addresses of welcome by the President and the Dean of St. Vincent College, a tour of the college buildings, the several meals and teas served in various college buildings, and a business meeting of the chapter.

The presiding officers for the meeting were Bernard L. Brinker, St. Vincent College, and Wheeler P. Davey, Pennsylvania State College.

Proceedings of the American Association of Physics Teachers

The Pasadena Meeting, June 17, 1941

THE fourth meeting of the American Association of Physics Teachers on the Pacific Coast was held in the Norman Bridge Laboratory of Physics, California Institute of Technology, Pasadena, on June 17, 1941, in connection with the annual meeting of the Pacific Coast Division, American Association for the Advancement of Science and Associated Societies. The program for the meeting was arranged by a committee consisting of Joseph Kaplan, S. W. Leifson, V. F. Lenzen, E. H. Warner, E. C. Watson, and W. Weniger.

Invited Papers

The afternoon session was devoted to the following invited papers:

A Demonstration of Critical Phenomena. Bruce H. Sage, *California Institute of Technology, Pasadena.*

Meteorology as a Physical Science. Jakob Bjerknes, *University of California, Los Angeles.*

Materials on the History of Science in the Huntington Library. Lyle Wright, *Henry E. Huntington Library and Art Gallery, San Marino.*

Aerodynamics in College Physics. Theodore von Kármán, *California Institute of Technology, Pasadena.*

Contributed Papers

Contributed papers were presented at the morning session as follows:

1. Physics Teaching as Related to Hydrology. N. W. Cummings, *Junior College, San Bernardino, Calif.*—Hydrology enables us to utilize known facts and principles in availing ourselves most advantageously of natural processes involving the movement and distribution of water on and near the earth's surface. It may turn out to be a useful proving ground in connection with the present tendency to try to make the methodology of physics contribute to the success of our democratic way of life. The process of evaporation, which is of importance in hydrology, may be used to illustrate this point. In many cases, the application of the principle of the conservation of energy takes on a particularly convenient form, and provides the easiest possible solution to the otherwise extremely difficult problem of determining losses from lakes. Nevertheless, when all the facts are examined, it becomes clear that certain popular beliefs regarding wind and humidity, which have dominated many studies of the problem, are incompatible with sound thermodynamics. This psychological phenomenon suggests more specifically how hydrology can be used as a proving ground for the wider applications of the scientific point of view as represented by physics. It focuses attention on the problem of getting people to accept and be governed by known facts.

2. A Laboratory Experiment on Newton's Second Law Illustrating the Scientific Method. E. H. Warner, *University of Arizona, Tucson.*—Most laboratory experiments do not teach the student how to determine the law for a given phenomenon; they simply permit verification of the law, which is either stated, or developed theoretically, before the experiment is performed. Yet it is the training in how to determine the equation describing the phenomenon that is the more useful and stimulating, especially if the student does not know, or cannot find in the textbook, the desired equation before performing the experiment. The simple experiment of releasing a uniform chain held at one end so that it hangs vertically with its lower end just touching the platform of a balance, and observing the balance reading at the instant the last link hits the balance platform, lends itself to such a procedure. With a series of chains of different lengths but of the same linear density, and of the same length but of different linear densities, it is possible to determine experimentally the equation that relates the maximum balance reading, the linear density and the length. The student should also be asked to compute from Newton's second law the maximum balance reading when a chain of weight W is allowed to fall upon the balance in the aforestated manner.

3. How Shall Geometrical Optics Be Taught? L. E. Dodd, *University of California, Los Angeles.*—Despite the need for a more widespread knowledge of geometrical optics—a highly analytical subject, indispensable in optical engineering and fundamental to optical instruments—the teaching of junior-senior courses in the subject has been neglected. The primary and continual emphasis should be on the *optics*, not the mathematics. It is practicable for the student to achieve a workable knowledge and appreciation without extensive use of higher mathematics, provided he has some facility with trigonometry and college algebra. This by no means encourages neglect of the mathematics, as the student will soon see. Practical suggestions were made for presenting the subject with enough attention to detailed problems and experiments to give real insight, with consequent interest, and considerable facility in analyzing, devising and setting up optical systems.

4. Performance of a High Precision Spherometer. Wayne Steimle and L. E. Dodd, *University of California, Los Angeles.*—Data were presented which were obtained with a highly precise imported spherometer of the Aldis type reading to approximately one-third the wave-length of sodium light. With this degree of sensitivity, it might be presupposed that an interferometric method of determining contact would be indispensable, which is not the case. Reproducibility, the working formula and practical questions pertaining to the use of a spherometer of this general type were discussed.

5. The Meaning of Dimensions. V. F. Lenzen, *University of California, Berkeley.*—It is still debated whether the

dimensions of a physical quantity express its nature or are arbitrary. The author proposes to resolve the conflict by distinguishing between the definition of a quantity and the dimensions of a unit. Definitions of a quantity in terms of fundamental quantities express the nature of that quantity. The word *dimension* should be reserved for the expression of the dependence of the change of a derived unit upon changes in fundamental units. For example, area may be defined as the product of two lengths. One may construct a system of units in which the unit of area is changed independently of the unit of length, and yet area may continue to be defined in terms of the product of two lengths. The view presented is related to the ideas of E. H. Kennard. The latter's concept of natural dimensions is criticized, but his concept of definitional dimensions recognizes the foregoing distinction between definitions and dimensions.

6. The *Tribuna di Galileo* in Florence. E. C. WATSON, *California Institute of Technology, Pasadena.*—This year marks the one-hundredth anniversary of the dedication of the *Tribuna di Galileo*, one of "the most evocative and inspiriting monuments ever raised to the memory of mortal man." The frescos in this exquisite memorial summarize in pictorial form and with rare simplicity and insight (1) the state of science before Galileo, (2) his own more important contributions, and (3) how his spirit and work lived on in the discoveries of his immediate successors and of succeeding generations. Lantern slides of these frescos were shown and were discussed in detail.

7. A Graphical Presentation of the Electron Theory of the Thermoelectric Effects. W. V. HOUSTON, *California Institute of Technology, Pasadena.*—By a study of the electron theory of the thermoelectric effects it is possible to isolate those features of the electron distribution that are responsible for the thermal electromotive force, the Peltier effect and the Thomson effect. It then becomes clear that these are features which appear only when a current exists and cannot be described in terms of the equilibrium distribution. It is then possible to plot the different parts of the statistical distribution, and to show graphically the reason for the various effects. It is also possible, on this basis, to make clear the distinction introduced by Bridgman between *driving electromotive force* and *working electromotive force*, and to give a concrete expression to these concepts.

8. On the Probability Concept of Statistical Mechanics. PAUL S. EPSTEIN, *California Institute of Technology, Pasadena.*—One of the most fruitful statistical concepts is Boltzmann's postulate that the probability of a state is measured by the number of ways in which this state can be realized. In classical statistics Boltzmann's method is applied to a gas or other system consisting of identical particles by counting the number of possible permutations of the particles that leave the state unchanged. In quantum statistics the legitimacy of regarding permutations as independent realizations of the state is denied. It seems at first sight that this fact plays havoc with Boltzmann's idea. However, a closer investigation shows that the concept can be made as useful in quantum statistics as in classical by distinguishing between the coarse and the fine definitions of a statistical state.

9. Concepts of Potential Difference and Electromotive Force as Presented in College Physics Textbooks. R. W. KENWORTHY, *University of Washington, Seattle.*—(Introduced by H. L. Brakel.) A study of college physics textbooks shows that the majority are vague, inaccurate, or incorrect in the presentation of the concepts of potential difference and electromotive force. Since the correctness of such concepts is essential to the understanding of the field of electricity, both student and teacher are placed at a serious disadvantage. The Association could render a distinct service to the teaching profession by having a committee that would give some form of approval only to textbooks which are free from such serious errors.

ATTENDANCE

The registration of those in attendance lists 19 members of the Association and 62 nonmembers. Members who registered were:

W. B. Anderson, Oregon State College; R. T. Birge, University of California; B. F. Boardman, Fresno State College; O. L. Brauer, San Jose State College; N. W. Cummings, San Bernardino Junior College; L. E. Dodd, University of California, Los Angeles; F. M. Durbin, Oklahoma A. & M. College; P. S. Epstein, California Institute of Technology; G. Foster, Pasadena Junior College; H. A. Kirkpatrick, Occidental College; P. Kirkpatrick, Stanford University; V. F. Lenzen, University of California; A. W. Nye, University of Southern California; G. T. Pelsor, University of New Mexico; C. B. Shaver, Pasadena Junior College; D. L. Soltau, Los Angeles; E. C. Watson, California Institute of Technology; W. Weniger, Oregon State College.

Government Positions Available

The U. S. Civil Service Commission, Washington, D. C., has announced openings in various grades of *inspector of naval ordnance materials*, with salaries of $1620 to $2600 a year. For some of the lower grades, undergraduate training in physics constitutes acceptable training and experience.

RECENT PUBLICATIONS AND TEACHING AIDS

Advanced Textbooks and Treatises

A Textbook of Sound. Ed. 2. A. B. Wood, Department of Scientific Research and Experiment, Admiralty, London. 594 p., 158 fig., 14 × 22 cm. *Macmillan*, $6.50. Published first in 1930 and now in revised and enlarged form, this textbook deals with vibrations of all frequencies, the emphasis being on modern theoretical and technical improvements as these are reported in the original papers. Many references are given to papers, particularly in cases where the literature of a new development is too voluminous to be condensed into textbook form. The treatment is divided into five main parts: theory of vibrations; vibrating systems and sources of sound; sound transmission; reception, transformation and measurement of sound energy; technical applications.

The Separation of Gases. M. Ruhemann. 296 p., 32 tables, 148 fig., 15×24 cm. *Oxford Univ. Press*, $5.75. During the 40 years that have elapsed since Linde first liquefied air on a commercial scale, workers in various countries—notably Russia—have gradually established low temperature gas separation, or "deep refrigeration," as an applied field of considerable technical importance. Indeed, the author of the present monograph predicts that in the near future all gases used in bulk commercially—even the atmosphere—will first be separated, at least partially, into a number of constituents, with a great gain in efficiency. Some of the important technical developments have not heretofore been described in English, but are now included in this monograph, which provides a comprehensive review of both the industrial and scientific aspects of the field. The book is one of The International Series of Monographs on Physics, the general editors of which are R. H. Fowler and P. Kapitza.

An Introduction to the Kinetic Theory of Gases. Sir James Jeans. 316 p., 31 fig., 14×22 cm. *Cambridge Univ. Press* and *Macmillan*, $3.50. Intended primarily to provide the knowledge of classical kinetic theory that is needed by the average student of physics or physical chemistry, this book contains most of the material that was of physical interest in the author's earlier treatise, *The dynamical theory of gases*, and omits much that was mainly of mathematical interest. However, the book is not merely an abbreviated edition of the earlier, more exhaustive work. Many of the statements have been remarkably simplified and clarified. Moreover, there is a new 50-page introductory chapter planned especially for the beginner in kinetic theory; it contains a brief history and illuminating preview of the methods and problems of the theory. Following this are the chapters on pressure, collisions, the free path, viscosity, heat conduction, diffusion, nonsteady states, statistical mechanics and thermodynamics, calorimetry and molecular structures. There are no lists of problems or questions.

An Introduction to the Theory of Newtonian Attraction. A. S. Ramsey, Fellow of Magdalene College, Cambridge. 193 p., many figures, 14×22 cm. *Cambridge Univ. Press* and *Macmillan*, $2.50. In the 18th and 19th centuries, the theory of gravitational attraction received much attention; nearly 1000 pages, the present author reminds us, were needed by Todhunter for his history of the subject. But today the theory of potential is studied in connection with the wider and practically more important field of electricity, and the subject of gravitational attraction, despite its importance in dynamical astronomy and the Figure of the Earth, tends to be overlooked. The present textbook on the subject, which is the last of a series on mechanics by this author [Am. J. Phys. **6**, 287(1938)], begins with a review of pertinent mathematical processes—surface and volume integrals, vector methods, fields, coordinate systems, and so forth—next discusses the fundamentals of the theory, and then treats internal points, the theorems of Laplace, Poisson, Gauss and Green, harmonic functions, and the attraction of ellipsoids. Numerous solved and unsolved examples are provided. The book does not have an index.

Radio-Frequency Measurements by Bridge and Resonance Methods. L. Hartshorn, The National Physical Laboratory (England). 277 p., 99 fig., 7 tables, 14×22 cm. *Wiley*, $4.50. Three kinds of quantities arise in the use of alternating currents of radiofrequency: (1) those, such as *current*, *potential difference* and *power*, that measure in various ways the strength of electric oscillations; (2) those, such as *impedance*, *capacitance*, *resistance* and *phase*, that depend on ratios of the quantities first named and measure in various ways the properties of circuits and the materials employed in them; and (3) *frequency*, which depends on time alone and is not essentially electric in character. The present book deals with measurements of the second group of quantities, the methods of which differ considerably from those used in other branches of electric measurements, but are of great importance both in radio precision testing and as research tools for investigating many electric, physico-chemical and biologic problems. The book starts as far as possible from first principles, and is confined to those methods that have proved to be most useful to the practicing technician. Special attention is given to such matters as screening, earth connections and sources of errors.

Phenomena at the Temperature of Liquid Helium. E. F. Burton, Director of the McLennan Laboratory, H. Grayson Smith, Associate Professor of Physics, and J. O.

WILHELM, Assistant Professor of Physics, University of Toronto. 373 p., 103 fig., 44 tables, 15×23 cm. *Reinhold*, $6. This authoritative reference work on low temperature physics is No. 83 of the American Chemical Society Monograph Series. The first third of the book provides an introduction to the methods of producing low temperatures, and of measuring these temperatures and the properties of matter involved; this part includes a survey of existing experimental results, and should be of interest to the general reader, as well as to those who seek an approach to the more modern phases of low temperature research. The remaining chapters, which contain the essence of the monograph, discuss modern theories of matter, particularly the solid and liquid states, in the light of low temperature phenomena—specific heats, magnetic phenomena, conductivities, superconductivity, and so forth. Although a few of the fundamental theoretical developments are described in detail, in most cases only the assumptions and final results are presented; but all this has been done quite skilfully, and with extensive references to papers and also to textbooks on theoretical physics.

Statistical Mechanics. JOSEPH EDWARD MAYER, Associate Professor of Chemistry, and MARIA GOEPPERT MAYER, Lecturer in Chemistry, Columbia University. 506 p., 28 fig., 12 tables, 15×23 cm. Wiley, $5.50. The aim of this textbook is to present statistical mechanics and its applications in a manner that will be understandable and useful to students of chemistry and physics whose familiarity with theoretical physics is limited. Although theoretical aspects are emphasized, very few experimental data being given, the whole treatment is relatively concrete and simple. Quantum statistics is introduced at the beginning; this is a most valuable feature, not only because the chemist's interest is largely in fields where quantum effects are important, but because the use of quantum language materially simplifies many of the considerations. The first four chapters contain an excellent account of ideal gas theory, a summary of the important equations of classical and quantum mechanics, and discussions and developments of the hypotheses and equations of both statistical mechanics and thermodynamics. The remaining 12 chapters deal with applications and include such topics as monatomic and polyatomic ideal gases, mixture of gases, ideal crystals, non-ideal gases, electric and magnetic fields, and modern statistics applied to radiation and to metals and degenerate gases. The senior author is the editor of *The Journal of Chemical Physics*.

Fundamental Processes of Electrical Discharge in Gases. LEONARD B. LOEB, Professor of Physics, University of California. 734 p., 297 fig., 49 tables, 15×23 cm. *Wiley*, $7. As is pointed out in the preface of this book, the study of electric discharge in gases was begun in the days of the discovery of x-rays and of the electron, and developed so rapidly that, by 1911 and 1914, J. J. Thomson and J. S. Townsend were able to summarize and correlate the many researches in their two classics, *Conduction of electricity through gases* and *Electricity in gases*. Because advances beyond this point were becoming increasingly difficult with the technics then existing, and also because other new and rich fields of research were opening up at the time, activity in the field of electric discharge in gases temporarily subsided, much of the further progress being left to the engineer. But the rapid development of atomic theory which began in 1926, coupled with remarkable improvements in technics and instruments—high vacuum, electron tubes, oscillographs—and an increasing interest in electronics in response to industrial needs, has led to a gradual revival of interest in the field. Since then a large number of specialized studies have been made, and the need has arisen for clarifying and interpreting their relations to one another and to the field as a whole, as well as for incorporating this new knowledge into handbooks and treatises. This is the main purpose of the present book; it treats the subject in the light of modern atomic physics, and at the same time connects the recent experimental results with the classical theory of the earlier works. Believing that an author should never ignore the controversial elements in his subject, PROFESSOR LOEB first presents the facts and the conflicting views; then, in the light of his extensive and pertinent experimental experience, expresses his own opinions, giving reasons in each instance. The subjects dealt with include ionic mobilities, recombination and diffusion of ions, electron mobility, distribution of electron energies in a gas in an electric field, the theory of probes, formation of negative ions, ionization currents, ionization by collision, the second Townsend coefficient, disruptive discharge in gases, the arc and glow discharges. The book is replete with useful and stimulating material. At the same time, it affords evidence of the author's seriousness in his contention, expressed in the preface, that "in writing a book of this scope . . . too much emphasis cannot be spared for style or for a detailed consideration as to the best organization in presenting material."

INTERMEDIATE TEXTBOOKS AND MANUALS

A Textbook of Heat. Part II. H. S. ALLEN and R. S. MAXWELL. 339 p., 193 fig., 14×22 cm. *Macmillan*, $3.50. The second volume of this work differs from the first [Am. J. Phys. **8**, 138(1940)] in that it is more theoretical and advanced in character. It starts with the laws, concepts and applications of thermodynamics, and after dealing with conduction, convection and the theory of radiation, passes on to an elementary and instructive treatment of statistical methods, probability and the quantum theory.

A Laboratory Course in Physics. N. F. SMITH, G. C. COMSTOCK and A. W. HANSON, The Citadel. 174 p., 102 fig., 21×28 cm. Lithoprinted, paper covers. *Edwards Brothers*, $2. Various departments offer a one-year, intermediate course in general physics that differs from the introductory course mainly in that it is more thorough and less descriptive. At The Citadel, however, the intermediate course differs in that it is essentially experimental in character, requiring three two-hour laboratory periods per week throughout the junior year. Two groups take the course—physics majors and premedical students—and all have

previously had a two-semester introductory course involving one period of laboratory work per week. The present manual, which was written especially for this intermediate course, contains directions, brief discussions of the theory and suitable literature references for 94 experiments. Each student performs approximately 70 experiments, the physics majors omitting topics covered in their other courses and the premedical students, much of the mechanics. Certain of the experiments, including some relatively simple ones on alternating-current circuits, have been designed especially for the premedical students. Calculus methods have been used wherever they seemed advisable, except in the experiments given solely to the premedical group.

Foundations of Modern Physics. THOMAS B. BROWN, Professor of Physics, The George Washington University. 345 p., 156 fig., 15×23 cm. *Wiley*, $3.25. This excellent textbook for a one-semester course in introductory modern physics emphasizes experimental aspects, the evidence given by experiment in support of current theories, and the most important practical outcomes of the period. Although the book does not follow the historical order in the development of each topic, it gives a considerable amount of space to those older parts of physics out of which recent advances have grown and which, on this account, really are integral parts of the modern development. The 18 chapters deal with such broad topics as the elementary charge, atomic dimensions, the wave character of light, polarized light, electric oscillations and electromagnetic waves, the electromagnetic spectrum, photons and electron waves, the hydrogen spectrum and atom, optical spectra and atomic structure, x-ray spectra, kinetic theory and molecular energy, crystals, temperature radiation, radioactivity, nuclear structure and theory, and cosmic rays. A discussion of restricted relativity appears in an 8-page appendix. Each chapter contains lists of problems and specific references to articles and books, all of which appear to have been selected with unusual care. The diagrams and photographs are especially helpful. Although the whole treatment is relatively simple and employs only elementary mathematics, it is genuinely instructive and certainly not superficial. Such a course following general physics could be taken with great profit by almost any college student of today.

General Physics for Students of Science. ROBERT B. LINDSAY, Professor of Physics, Brown University. 548 p., 265 fig., 15×23 cm. *Wiley*, $3.75. Thorough, rigorous, and effective in its emphasis on physical methodology and the evolution of theories, this noteworthy textbook provides a course of introductory or intermediate grade for the serious science student who has had mathematics through elementary calculus. Although restricted to material that is fundamental for the student in subsequent work in any of the sciences, the treatment nevertheless is unusually comprehensive; this is largely because the mathematical methods that are natural and desirable for the development of each topic have been employed, thus making it possible to simplify and shorten the essential developments. Considerable space thus is freed for the valuable and stimulating discussions of meanings and implications that characterize the work, and perhaps make it most distinctive and important. The book opens with a 29-page discussion of symbols, laws, and theories, general properties of matter, and a brief history of the field. The subsequent order of development is, briefly, as follows: *Part II*, particle dynamics and statics, rigid bodies, elasticity, fluids; *Part III*, temperature and thermal expansion, calorimetry and change of state, kinetic theory, thermodynamics, heat transfer; *Part IV*, magnetostatics, electrostatics, steady currents, thermoelectricity, electromagnetism, alternating currents, discharge in gases; *Part V*, wave motion and acoustics, geometrical optics, physical optics, radiation and the constitution of matter. An extensive list of problems is provided for each chapter. This book doubtless will give impetus to the present healthy tendency to distinguish sharply between courses intended for the general student and those planned for students who have the desire and the ability to enter science as a profession.

FIRST-YEAR COLLEGE PHYSICS

An Introductory Course in College Physics. N. HENRY BLACK, Assistant Professor Emeritus of Physics, Harvard University. Rev. ed., 742 p., many figures, 15×24 cm. *Macmillan*, $3.75. In carrying out this thorough revision, the author has retained the many good features of the original edition [Am. J. Phys. **3**, 139 (1935)] and yet has made numerous improvements. Increased attention has been given to the needs and interests, first, of premedical students, and then of pre-engineering and liberal arts students, for these three groups, in the order named, have been found to predominate in the introductory course in Harvard College. The schematic diagrams and photographs have been revised to advantage. The numerous problems, now supplied with answers in many cases, are graded in difficulty and designed so as to avoid mere arithmetic drudgery.

Household Physics Laboratory Manual. MADALYN AVERY, Professor of Physics, Kansas State College of Agriculture and Applied Science. 99 p., illustrated, 22×28 cm. *Macmillan*, paper cover, $1.50. Designed to accompany the author's textbook, *Household Physics* [Am. J. Phys. **7**, 75(1939)], this loose-leaf manual contains the theories, procedures, questions and blank data sheets for 25 quantitative laboratory experiments. Although these experiments are essentially standard in character, they have been adapted to the needs of home economics students, a number of them dealing with such topics as household measurements, breaking strengths of textiles, vacuum cleaners, fuels, insulating properties of materials, household motors, electric heaters, illumination, and the optics of the eye.

College Physics—Abridged. HENRY A. PERKINS, Professor of Physics, Trinity College, 600 p., 470 fig. and plates, 15×23 cm. *Prentice-Hall*, $3.50. Although this abridged edition of the author's *College Physics* [Am. J. Phys. **7**, 75(1939)] has been shortened by some 200 pages, the reduction has been wisely accomplished, not by re-

sorting to condensation, but by eliminating many topics of relatively small importance and a few that are especially difficult. Thus the new version retains the admirable features of the original edition, such as the emphasis on fundamental principles and the careful detailed treatments of the topics covered; yet it is more suitable for use in a one-year course intended primarily for nontechnical students. Many statements have been rewritten in the interest of greater clarity and simplicity. The treatment of units in mechanics has been somewhat modified.

Demonstration Lectures in Physics—Lecture Outline and Record Sheets. WILLIAM S. WEBB, Professor of Physics, and Bertrand P. Ramsay, Assistant Professor of Physics, University of Kentucky. 90 4-p. leaflets, 15×28 cm. *Central Scientific Co.*, 3 cts. per leaflet. Beginning in 1926, Professor Webb and his associates at the University of Kentucky have developed a remarkably successful cultural course in physics which at the present time is taken as an elective by approximately one-fourth of the students in the University. The course is built on lecture demonstrations as the central feature, and at the beginning of each of these lectures the student is given a leaflet that contains a synopsis of the topics to be covered, lists of reading references and questions, a page of photographs and diagrams illustrating various apparatus and phenomena to be dealt with, and a blank page for notes and sketches. These interesting leaflets for the 90 lectures that comprise the year's course are now available in an attractive loose-leaf form, and may be purchased by any institution at cost —2.4 cts. per leaflet. The lectures are divided into ten main groups—on foundations of physics, mechanics, fluids, heat, wave motion and sound, electrostatics and magnetism, direct currents, alternating currents, optics, light, and matter. At the University of Kentucky the first semester of the course is not a prerequisite for the second. A pamphlet on the management of such a course may be secured from the publisher.

COLLEGE SURVEY COURSES

The Physical Sciences. EMMETT JAMES CABLE, ROBERT WARD GETCHELL and WILLIAM HENRY KADESCH. 771 p., 300 fig., 15×23 cm. *Prentice-Hall*, $3.75 (school), $5 (trade). The authors—a geographer, a chemist and a physicist, respectively—have selected the material for this survey textbook with intelligent regard for the needs of the modern, general student. The subject matter of the first half of the book (315 p.) is drawn from the field of physics, including meteorology; the remainder is from chemistry (143 p.), geology (80 p.) and astronomy (101 p.). The treatment is qualitative and descriptive in character, is for the most part neither loose nor misleading, and is quite coherent; the various chapters are not mere unrelated fragments. Lists of questions and references for collateral reading have been supplied for each chapter. The book work is excellent.

A Survey of Physical Science. LINVILLE L. HENDREN, Professor of Physics and Astronomy, University of Georgia. 566 p., 287 fig., 14×21 cm. *Univ. of Georgia Press*, $3.25. The colleges that constitute the University System of Georgia offer a survey course that covers the sciences of physics, astronomy, chemistry and geology. The present textbook is based on the syllabus for the first half of this course—the equivalent of 3 semester-hours—and is intended to provide an elementary survey of the more fundamental aspects and main practical applications of physics and astronomy, and to give some idea of the meaning and value of scientific methods and how physical science has affected our ways of thinking and manner of living. Most of the physics is covered in the first 70 percent of the book. Numerous specific references are given to 20 books of various types which comprise a basic reference set for the course. There are many solved, quantitative examples, and also lists of questions and simple problems. The physics portion of this textbook is also available in a separate volume, under the title, *Survey of Elementary Physics* (403 p., 232 fig., $2.25); it is used at the University of Georgia in a one-quarter elementary course of 4 lectures and one laboratory period each week.

BOOKS FOR SECONDARY AND ELEMENTARY SCHOOLS

Physics for Secondary Schools. OSCAR M. STEWART, Professor of Physics, University of Missouri, and BURTON L. CUSHING, East Boston High School and Harvard Graduate School of Education. Rev. ed. 768 p., 564 fig. and plates, 13×20 cm. *Ginn*, $1.80. Appearing originally in 1932 under the authorship of Stewart, Cushing and Towne, this successful textbook for a senior high school physics course has now been revised to include new material on recent applications and developments such as television, fluorescent lamps, induced radioactivity and the cyclotron. New simple problems have been added.

Everyday Problems in Science. W. L. BEAUCHAMP, University of Chicago; JOHN C. MAYFIELD, University High School, Chicago; and JOE YOUNG WEST, Maryland State Teachers College. Ed. 3. 768 p., 532 fig., 20 tables, 15×23 cm. *Scott, Foresman*, $1.72. This textbook and manual for a first-year high school course in general science is carefully planned and interestingly written. Approximately two thirds of the material is taken from the physical sciences. The book work and the illustrations are modern and excellent. Available for optional use with the book is a cleverly designed assignment-and-study book of the self-help type (346 p., 21×27 cm; paper cover, 84 cts.).

What Makes the Wheels Go Round—A First-Time Physics. EDWARD G. HUEY, Calvert School, Baltimore. 187 p., many illustrations, 19×25 cm. *Reynal & Hitchcock*, $2.50. This splendid book represents probably as successful an attempt as has yet been made to provide children with an effective reading book or textbook in physics. It lacks arithmetic, exercises to solve, and even required experiments. But it is not superficial; the material in it is for the most part "good physics," expertly simplified by a teacher who has a sense for the many things about physical nature and science—some of them genuinely fundamental—that a child living in this day should learn long before he reaches

the secondary school or college. The format, typography and illustrations are appropriate and attractive.

Methodology and Philosophy of Science

Procedures and Metaphysics. Edward W. Strong. 308 p., 2 fig., 13×21 cm. *Univ. of California Press*, $2.50. A critical study of the procedures of mathematical-physical science in the sixteenth and seventeenth centuries, and of the metaphysical structures that grew out of them.

The Kantian Philosophy of Space. Christopher Browne Garnett, Jr., Associate Professor of Philosophy, George Washington University. 298 p., 15×23 cm. *Columbia Univ. Press*, $3.50. Analyzing critically the Kantian doctrines of space as they develop chronologically in Kant's own writings, the author seeks to display the various tendencies in Kant's views and to determine their relative importance in his general philosophy and also in relation to present-day philosophy and theoretical physics. The study is prefaced with an 80-page discussion of the views on space held by Newton, Leibnitz and Samuel Clarke, and concludes with a chapter on the relation of the Kantian doctrines to the contemporary views of Alexander, Whitehead and Broad.

History of Physics

Giovanni Marliani and Late Medieval Physics. Marshall Clagett. 182 p. *Columbia Univ. Press*, $2.50. The author has provided a documented study and sober evaluation of certain fourteenth and fifteenth century problems and concepts in heat, mechanics, and mathematics. Although the physicist Marliani (d. 1483) made little progress beyond his English and French precursors, he was superior to most of his contemporaries and provided records of the physics of his time that are of considerable value to the student of medieval thought.

Portraits of Famous Philosophers Who Were Also Mathematicians. With biographical accounts by Cassius Jackson Keyser, Adrian Professor Emeritus of Mathematics, Columbia University. *Scripta Mathematica* (Yeshiva College, N. Y.), $3. Similar in character and quality to the portfolios of eminent mathematicians published by *Scripta Mathematica* [Am. J. Phys. **4**, 216 (1936); **7**, 425 (1938)], this volume contains reproductions of portraits of Pythagoras, Plato, Aristotle, Epicurus, Roger Bacon, Descartes, Pascal, Spinoza, Leibnitz, Berkeley, Kant and Charles Sanders Peirce. Since no genuine effigies of Pythagoras and Bacon are in existence, the pictures of them necessarily represent only conceptions of the artists who made them. In the case of Plato, Aristotle and Epicurus, the pictures are taken from ancient statues which are believed by some authorities to be genuine portraits.

Engineering

The Coordination of Engineering Curricula. Proceedings of the Sixth Annual Meeting of the Allegheny Section, Society for the Promotion of Engineering Education. 89 p., 15×23 cm. *Carnegie Institute of Technology*, paper cover, 50 cts. Seven papers, with discussion, on engineering education—its fundamentals, its scientific, humanistic and social aspects, its coordination with industry and with the liberal arts college, and so forth.

Mechanism and the Kinematics of Machines. W. Steeds, Lecturer in Mechanical Engineering, The Military College of Science. 330 p., 379 fig., 14×22 cm. *Longmans, Green*, $5. Books on the theory of machines usually include statics and kinetics, in which case the kinematics is slighted, or else they deal with kinematics alone, but in a manner too comprehensive and detailed to be of much use to the student. But the present book follows a middle course, intended to make it useful either as a textbook for engineering colleges or as a reference work for engineers, designers, and draftsmen. Topics that have received unusual emphasis include the freedom and constraint of bodies, geometric design, velocity and acceleration of points, the Coriolis law, and the theory of toothed gearing.

An Introduction to the Study of Air Mass and Isentropic Analysis. Jerome Namias and others. Ed. by Robert G. Stone, 238 p., many figures, 15×23 cm. *American Meteorological Soc.* (Milton, Mass.), paper cover, $1.25. Published in previous editions under the title, *An Introduction to the Study of Air Mass Analysis* [Am. J. Phys. **5**, 94 (1937)], this monograph now appears as a reprint of the text of the fourth edition (1938), but with numerous annotations and alterations, and the addition of a chapter on isentropic analysis. The bibliography has also been enlarged, and is arranged conveniently by subjects. The monograph is not intended as a textbook of synoptic meteorology, but as an adjunct to existing textbooks of certain new topics that are not particularly well treated outside of a few technical and expensive works.

Tables and Nomograms

Atlas of Nomograms for Physical Chemistry. G. V. Vinogradov and A. I. Krasitlschikov. 33×44 cm. *U.S.S.R. Society for Cultural Relations with Foreign Countries* (Moscow, 56). Representing the first attempt ever made to systematize the application of nomography to the study of physico-chemical problems, this atlas of 200 nomograms aims at showing the appearance and great utility of the alignment charts for various well-known formulas in the domains of ideal gases and kinetic theory, vapor pressure, absorption and surface phenomena, theory of dipole moments, electrochemistry, solutions, and the physics of hydro- and aero-sols. Accompanying the atlas is a 144-page volume, in Russian, on the principles of nomographic calculations and the use of nomograms.

Mathematical Tables. Herbert Bristol Dwight, Professor of Electrical Machinery, Massachusetts Institute of Technology. 238 p., 15×23 cm. *McGraw-Hill*, lithoprinted, cloth bound, $2.50. The tables included are: trigonometric

functions of decimals of degrees and logarithms, natural and common logarithms; the exponential function; hyperbolic functions; binomial coefficients; factorials; Gregory-Newton interpolation coefficients; Lagrange interpolation coefficients for interpolating without columns of differences; surface zonal harmonics and their first derivatives; complete elliptic integrals of the first and second kinds; Bernoulli numbers; Euler numbers; gamma function; error function; Bessel functions; Riemann zeta function. For practically all of the tables differences are given, and most of them are small so that interpolation can be accomplished with a slide rule or even mentally.

Miscellaneous Books

Time and Its Mysteries. Series II. 147 p., 4 fig., 14×22 cm. *New York Univ. Press*, $2. The four nontechnical lectures appearing in this volume comprise the second series on time and timepieces given at New York University on the Arthur Foundation [Am. J. Phys. **5**, 238 (1937)]. The titles are: *The time concept and time sense*, by the late D. W. Hering; *What is time?*, by W. F. G. Swann; *Time and individuality*, by John Dewey; *Time and the growth of physics*, by A. H. Compton.

Education on an International Scale. George W. Gray. Introduction by Raymond B. Fosdick. 128 p., 14×21 cm. *Harcourt-Brace*, $2. This is a history of the International Education Board, 1923–1938, a foundation whose work, now finished, has involved appropriations ranging from a few hundreds of dollars to the six millions given for the construction of the 200-in. telescope. "For a total expenditure of something less than half the cost of a battleship, work was accomplished in the sciences, in agriculture, and in the humanities which even war cannot destroy."

The Birth and Death of the Sun. George Gamow, Professor of Theoretical Physics, George Washington University. 252 p., 76 fig. and plates, 15×22 cm. *Viking Press*, $3. This dramatic and remarkably simplified account of stellar evolution and subatomic energy may be highly recommended to the general reader and to beginners in physical science. The illustrations, which include schematic drawings by the author, are effective. The book work is excellent.

The Chemical Action of Ultraviolet Rays. Carleton Ellis and Alfred A. Wells. Revised and enlarged edition by Francis F. Heyroth, University of Cincinnati. 970 p., 159 fig., 15×23 cm. *Reinhold*, $12. The two senior authors' familiar reference work on photochemistry is here presented in a thoroughly revised and enlarged edition. The book is replete with detailed information on the sources of ultraviolet radiations, photochemical processes, applications of photochemistry to industrial processes and applications of ultraviolet radiations in biology. Specific references are given to the voluminous literature from which the material is drawn. This comprehensive reference work should be of especial value to workers in theoretical and industrial chemistry, biology and medicine.

Odd Numbers, or Arithmetic Revisited. Herbert McKay. 215 p., 13×20 cm. *Cambridge Univ. Press* and *Macmillan*, $2.50. That the understanding and manipulation of numbers is interesting and imaginative for its own sake is the theme of this entertaining and informative little book for the layman. With the help of numerous worked examples of considerable novelty and practical interest, the author clarifies the meanings and uses of large numbers, exponents, proportion, weights and measures, "the delusive average," approximations and so forth.

The Musical Ear. Ll. S. Lloyd, Department of Scientific and Industrial Research (England). 87 p., 29 fig., 14×22 cm. *Oxford Univ. Press*, $1.50. Written for the intelligent musician and assuming no previous knowledge of science, the seven essays in this interesting little book reflect the point of view that, for an understanding of musical acoustics, one must start with correct musical premises; that musicians are rightly impatient with musical acoustics when it conflicts with the art and practice of music; that the fault lies with "theoreticians" who have ignored the problems of hearing. The titles of the essays are: Intonation; Electronic organs and the phonodeik; Helmholtz; The sounds of church bells; The notes of the harmonic series; The sounds of distant music; The scale and the musical ear. The book lacks an index.

Pamphlets and Reprints

Bell Telephone System Monographs. *Bell Telephone Laboratories* (463 West St., New York), gratis to college departments. K. K. Darrow, *Nuclear fission*, B-1218; K. K. Darrow, *Analysis of the ionosphere*, B-1241; K. K. Darrow, *Helium the superfluid*, B-1247; H. Dudley, *The carrier nature of speech*, B-1254; J. C. Steinberg, H. C. Montgomery, M. B. Gardner, *Results of the World's Fair hearing tests*, B-1256; C. R. Englund, A. B. Crawford, W. W. Mumford, *Ultra-short-wave transmission over a 39-mile "optical" path*, B-1259; W. E. Jackson, A. Alford, P. F. Byrne, H. B. Fischer, *The development of the Civil Aeronautics Authority instrument landing system*, B-1280; K. K. Darrow, *Debt of modern physics to recent instruments*, B-1281; O. J. Murphy, *Measurements of orchestral pitch*, B-1282.

Graduate Study and Research in Physics and Astronomy. 15 p., 6 fig. *Graduate School, Ohio State University*, gratis. An attractive booklet that describes the opportunities for graduate study and research in this department.

The Rockefeller Foundation—A Review for 1940. Raymond B. Fosdick. 64 p., 18 fig. *Rockefeller Foundation* (49 W. 49th St., New York), gratis to college teachers. Every physicist should read the section on "Science and the Moral Order."

Motion Picture Films

World's Greatest Telescope Reflector. 16 mm, sound or silent, 11 min. *California Institute of Technology*, rental or sale. Grinding of the 200-in. reflector.

Scientists Speak for Peace and Democracy. 16 mm, sound, 10 min. *Bell and Howell Co.* (1801 Larchmont Ave., Chicago), rental or sale. Talks by Einstein, Urey, Thomas Mann and A. H. Compton.

The Eyes—Advanced. 16 or 35 mm, silent, *Eastman Teaching Films* (Rochester), sale only. Anatomy and dissection of eye; eye-glasses; eye hygiene.

The Automobile. 16 or 35 mm, sound, 7 to 10 min. *Jim Handy Organization* (2821 East Grand Blvd., Detroit), sale only. Prepared for purposes of national defense, each of the 18 reels in this series deals with some aspect of the automobile such as engine operation, cylinder head, lubrication, ignition, clutch, gears, differential, steering geer, brakes, etc.

Elementary Operations on the Engine Lathe. 16 mm, sound, 11 min. *Eastman Teaching Films* (Rochester), sale only. Methods for exact alignment of the lathe; detailed procedures in facing, straight turning and squaring a shoulder; working accurately to dimensions from a mechanical drawing; characteristics and selection of cutting tools; use of micrometers.

Spark Plugs in Aviation. 16 and 35 mm, sound, 30 min. *Bray Studios* (729 Seventh Ave., New York), rental or sale. Construction, operation and testing of aviation spark plugs; ignition troubles.

How to Make Good Movies. 16 mm, silent. *Harmon Foundation* (140 Nassau St., New York), rental, $1.50 per reel. New films in this series [Am. J. Phys. **8**, 329(1940)] are (5) *How to use filters* and (6) *Lenses and their uses.*

TVA. 16 or 35 mm, sound, 20 min. *TVA Information Office* (Knoxville, Tenn.), loaned gratis. Dam construction, rural electrification, etc.

Slidefilms

Educational Slidefilms. 35 mm. *Jim Handy Organization* (2821 East Grand Blvd., Detroit), sale only. (1) *Electricity*, a course of 12 slidefilms on all common phases and applications of the subject, suitable especially for high schools; (2) *Automobile Training Course*, a course of 35 slidefilms dealing with all parts of an automobile; (3) *Pilot Training Course*, 24 slidefilms on all phases of ground and flying instruction.

DIGEST OF PERIODICAL LITERATURE

Terminology and Units

Cycles per second. Anon., *J. Sci. Inst.* **18**, 51 (1941). The growing tendency to express frequencies in *cycles*, instead of in *cycles per second*, is to be deplored as slovenly and incorrect. *Frequency* implies a time dimension, and there is no justification for ignoring the "per second" here, any more than in, say, a velocity.—H. N. O.

The mil as an angular unit and its importance to the Army. R. S. Burlington; *Am. Math. Mo.* **48**, 188 (1941). In the Army two systems for measuring angles are used, the mil system whose unit, the *mil*, is 1/1600 of a right angle, and the common sexagesimal system whose unit is the *degree*. The sight scales, azimuth circles and quadrants of most American mobile artillery units and of many heavy railway mounts are calibrated in mils. The fact that the mil is approximately the angle subtended by 1 yd at a distance of 1000 yd makes the unit well adapted to certain types of rapid calculations and accounts for its extensive military use. The Sub-Committee on Education for Service of the War Preparedness Committee (mathematics) recommends that trigonometry teachers add this system to their course work.—H. N. O.

Speed of Light

Early estimates of the speed of light. C. B. Boyer; *Isis* **33**, 24–40 (1941). An astonishing degree of uncertainty and inconsistency exists concerning the early estimates of the speed of light. In 1671–75 Roemer, Picard and Cassini observed the eclipses of the satellites of Jupiter. Roemer found that the "second inequality" in the recurrence of the eclipses of the first satellite could be explained on the hypothesis that light is propagated with a finite and determinable speed c. The closest that Roemer came to giving an explicit value of c was to say that light requires less than 1 sec of time to traverse a distance of about 3000 leagues. His estimate for c must be deduced from his conclusion, based on observations of the first satellite of Jupiter, that light requires about 22 min to traverse the orbit of the earth. If Roemer had accepted Cassini's round figure of 91,000,000 mi for the distance of the sun from the earth, he might have inferred a speed of about 138,000 mi sec^{-1}; if he had accepted Picard's degree and Cassini's deduction from it of a distance of 86,000,000 mi, he could have obtained 130,000 mi sec^{-1}.

However, values ascribed to Roemer in the current literature vary from 120,000 to 200,000 mi sec^{-1}; 17

different values were obtained in an examination of about 30 standard textbooks, histories and articles. "The erroneous value of 192,000 mi sec^{-1} or its equivalent . . . is so frequently imputed to Roemer that it threatens to become traditional." Most of these discrepancies are due to the use of modern data in calculations by Roemer's method.

The values given for Roemer's measurement of the time taken to cross the earth's orbit—a value that is readily available in source material—vary nearly as much as that for the computed speed. Fourteen works, including standard textbooks in various languages, give seven different values, none of them near the correct one of 22 sec. It is possible that Roemer made a better estimate of the time than 22 sec, but since many of his manuscripts were burned in 1728, this cannot be verified.

Huygens deduced from Roemer's work that the speed of light was 600,000 times that of sound. In 1669 Huygens set the speed of sound at 180 toise/sec. Again a number of textbooks give values other than that of Huygens.

As early as 1675, Cassini would seem to have given the time of transit of light from the sun to the earth as 10 or 11 min. In 1693 he gave 14 min, 10 sec as "the greatest correction in the case of the second inequality of the first satellite of Jupiter," that is, as the time for light to cross the earth's orbit. Halley gives 14 min, which would (with the value Halley accepted for the sun's parallax) lead to a value for c of about 155,000 mi sec^{-1}.

Newton gives the time of transit from the sun to the earth as 10 min, in the first edition of the *Principia*, and as 7 or 8 min in the *Opticks* and in later editions of the *Principia*. This last value was used by Newton to compute c, which he gave as 700,000 times the speed of sound. The speed of sound he thought to be "1140 English feet per second minute of time." The *Opticks* implies that Roemer determined the time of transit from the sun as 7 or 8 min, which may have contributed to the confusion that is found throughout the period since then. Both the *Encyclopédie* and Priestley's *History* ascribe this value to Roemer.

Bradley found an angle of astronomical aberration of about 20″.2, from which he computed that c is 10,210 times the earth's orbital speed. He did not give a value for c, although a number of textbooks ascribe several different values to him, but did give 8 min and 12 or 13 sec for the retardation of the light of the sun. He appears to have used 10″1/3 for the solar parallax, which corresponds to a distance of 79,100,000 mi and yields a value of 161,000 mi sec^{-1} for c.

Whether because of the lack of a satisfactory theory of light or because of the practical difficulties inherent in the astronomical procedures, very little interest in the precise determination of c appears to have been manifested before the middle of the nineteenth century.—D. H. D. R.

Check List of Periodical Literature

Science programs for adult radio listeners. A. L. Elder, K. G. Bartlett; *J. Chem. Ed.* **18**, 190–192 (1941). Useful information on how to conduct programs.

Fast method for finding areas or mean ordinates of curves. A. D. Moore; *J. Eng. Ed.* **31**, 452–459 (1941). For many purposes the method is more simple, rapid and accurate than the use of the planimeter, Simpson's rule or the counting squares.

Problems in secondary education that affect engineering colleges. F. B. Lindsay; *J. Eng. Ed.* **31**, 479–486 (1941).

Count Rumford. W. J. Sparrow; *Sch. Sci. Rev.* **22**, 280–287 (1941). A brief account of his life and work, calling attention to his wide scientific and philanthropic interests.

Applied nuclear physics. R. D. Evans, H. C. Urey, G. Failla; *J. App. Phys.* **12**, 259–295 (1941). Three articles on (1) applications of nuclear physics in chemistry, geology, physiology and medicine, (2) the four main methods of separating isotopes, and (3) the biological effects of ionizing radiations.

Soil dynamics. C. C. Nikiforoff; *Sci. Mo.* **52**, 422–429 (1941). A brief survey of important ideas and problems involved in soil physics and chemistry.

The physicist and evolving civilization. L. Tonks; *Sci. Mo.* **52**, 430–438 (1941). The general historical relation of science to society, and some of its contemporary social consequences.

The elimination of hypo from photographic images. J. I. Crabtree, G. T. Eaton, L. E. Muehler; *J. Frank. Inst.* **230**, 701–725 (1940).

The velocity of sound. R. C. Colwell, A. W. Friend, L. H. Gibson; *J. Frank. Inst.* **230**, 749–754 (1940). An electrical method applicable to small distances in air.

A simple method of building close-packed molecular and crystal models. E. A. Hauser; *J. Chem. Ed.* **18**, 164–166 (1941). By using balls of any convenient material which have been coated with unvulcanized rubber, true magnified copies of structures can be prepared rapidly and inexpensively.

The probability concept. E. C. Kemble; *Phil. of Sci.* **8**, 204–233 (1941). In the author's opinion the controversy over the meaning of *probability* is resolved by the acceptance of the idea that the word has not one but three meanings: (1) a priori and (2) inductive probabilities, defined by the way they are derived from experience, and (3) theoretical probabilities that are constructs defined by the way in which they are to be compared with experimental data. The general approach is Bridgman's operational procedure.

Note on Liouville's theorem and Heisenberg's uncertainty principle. J. H. Van Vleck; *Phil. of Sci.* **8**, 275–279 (1941). An analytic explanation of an apparent paradox.

Anti-reflection films on glass. A. F. Turner; *Educ. Focus* (Bausch & Lomb), Spring, 1941. Elementary, but good.

The sequence illusion. R. L. Ives; *J. Frank. Inst.* **230**, 755–763 (1940). The physical and physiological aspects of various sequence illusions; for example, the report of a casual observer that a long line of similar boulevard lamps which are lighted simultaneously appear to light in sequence, the nearest lamp first, with the time interval between successive illuminations progressively becoming smaller.

Geiger-Mueller counters. C. G. and D. D. Montgomery; *J. Frank. Inst.* **231**, 447–467 (1941). A review of methods of construction, applications; what the empirical procedures accomplish and how they are justified.